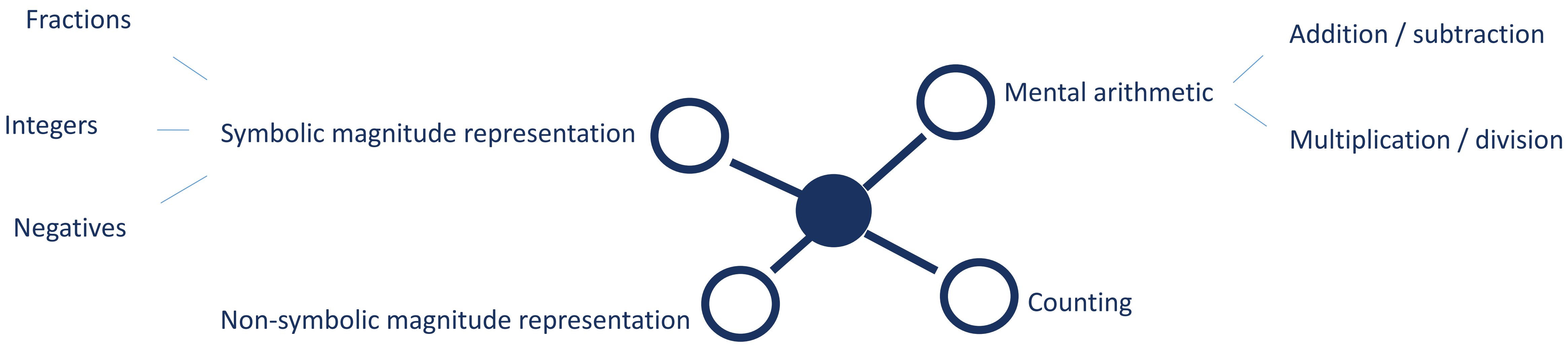
Space: The Loom of (Numerical) Cognition

André Knops

Marseille 11.06.2026

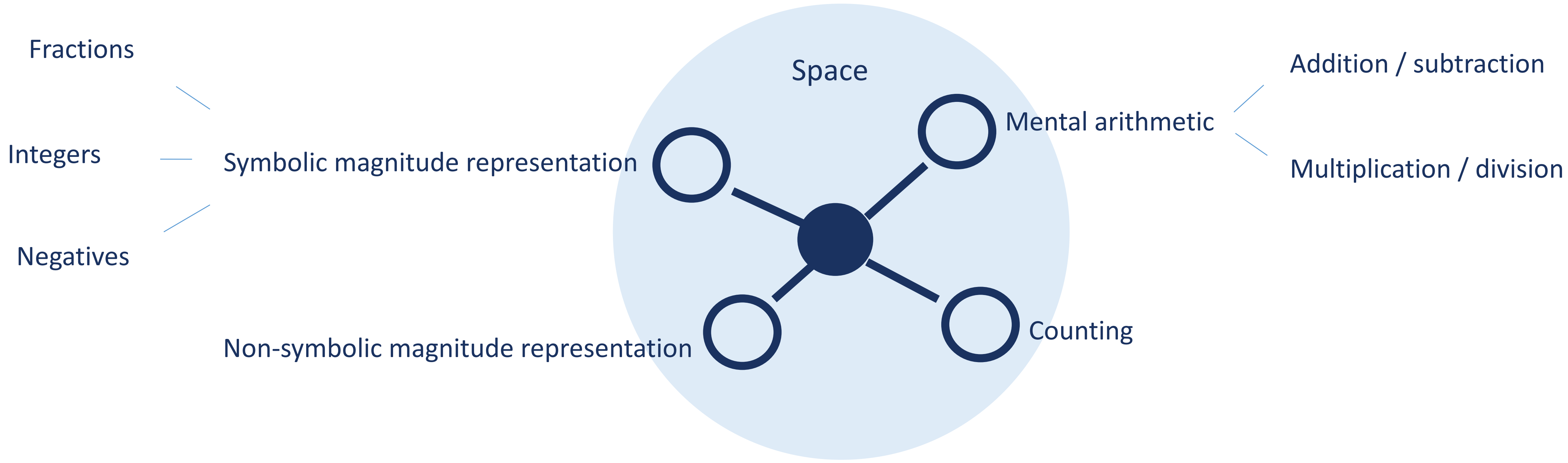
Université Paris Cité

Numerical cognition has many facets



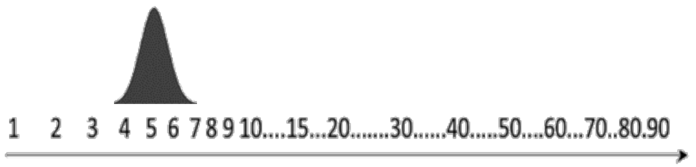
No theoretical consensus for any of these facets

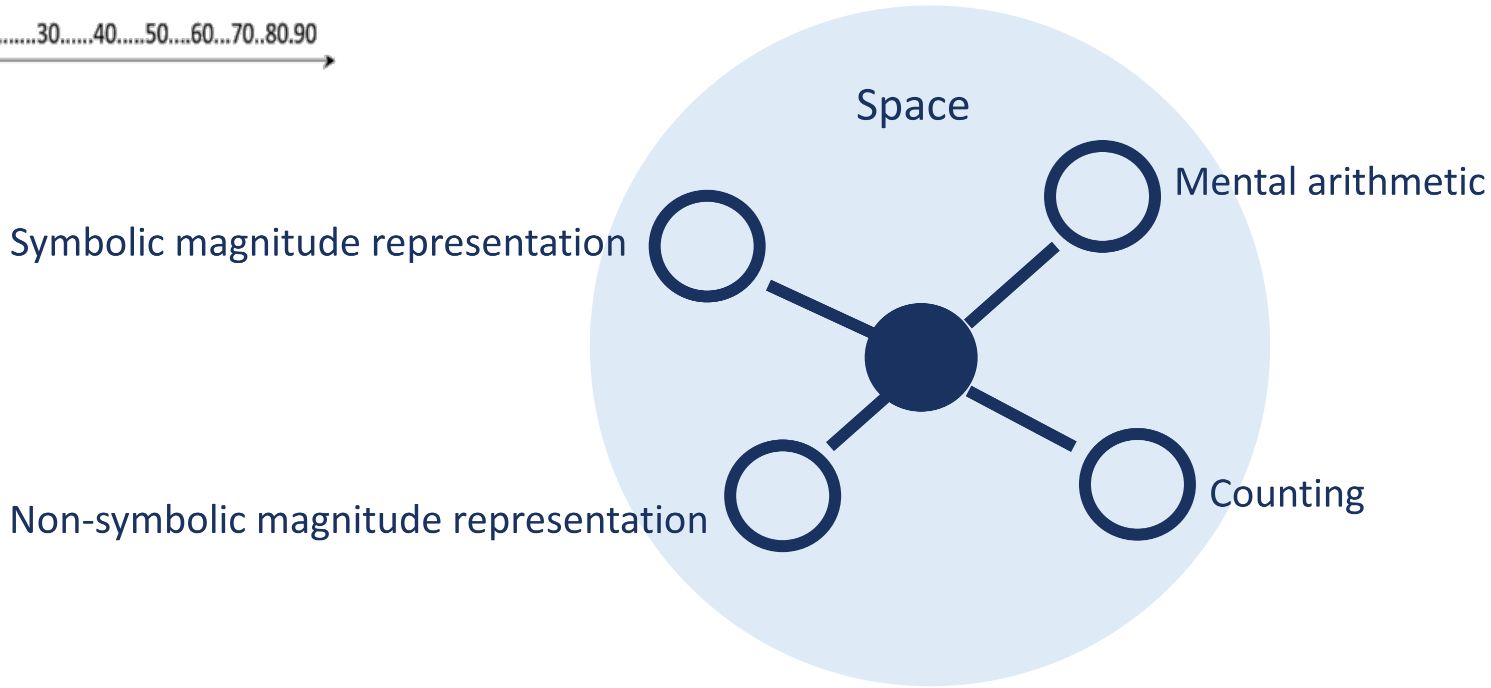
Numerical cognition has many facets



Spatial processes contribute to these functions

Numerical cognition has many facets



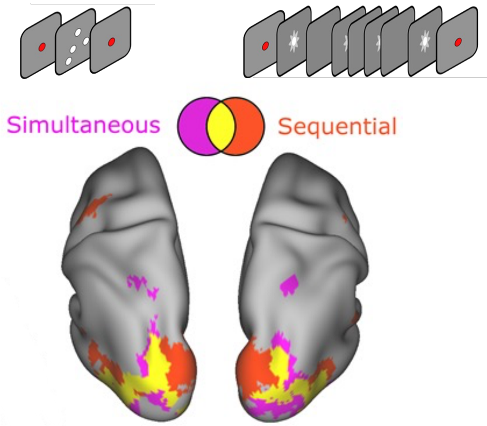


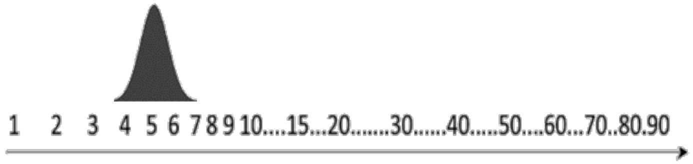
Spatial processes contribute to these functions

Neurocognitive **Architecture** of Numerical Information

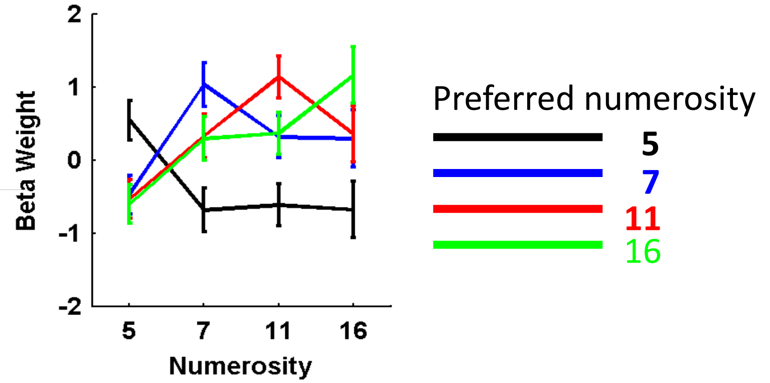
- Numerical magnitude is specific to
  - Mode (simulatenous / sequential)
  - Modality (visual / tactile)

My results





Simultaneous



Sequential

No number tuning observed

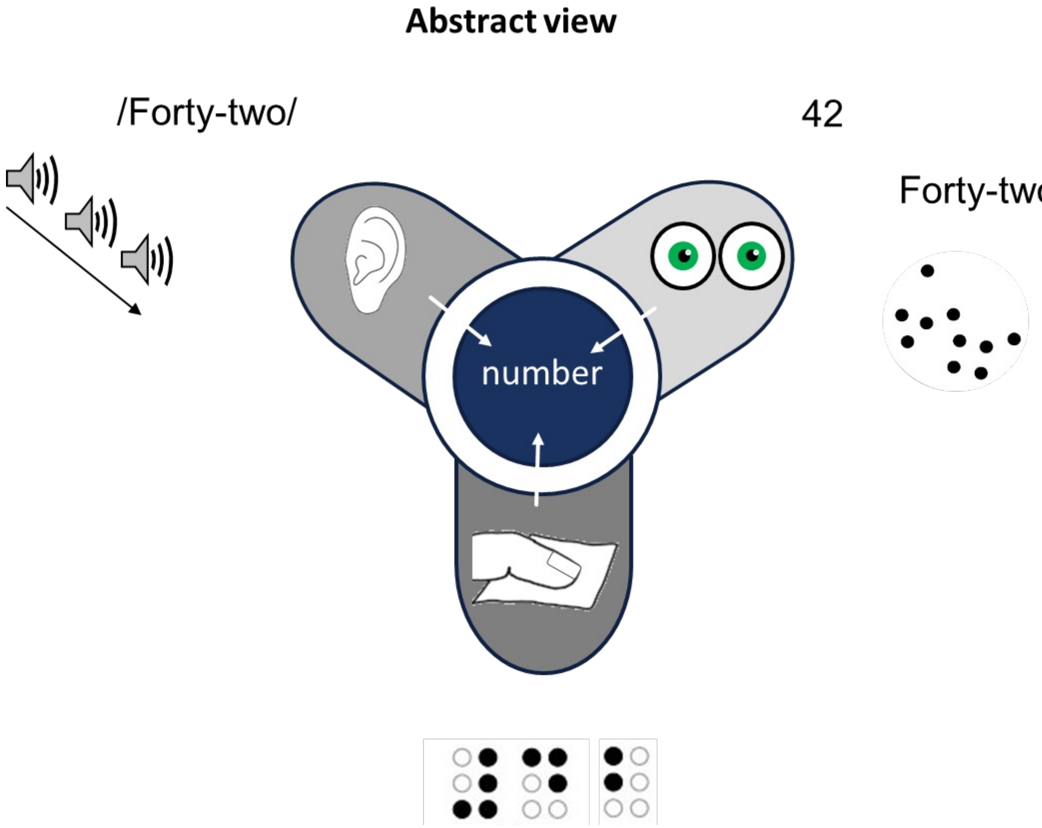
For simultaneous numerosities, parietal cortex shows number selectivity that is absent for sequential numerosities

Cavdaroglu et al., *Cereb Cortex* 2019

Neurocognitive **Architecture** of Numerical Information

- Numerical magnitude is specific to
  - Mode (simulatenous / sequential)
  - Modality (visual / tactile)

Hypothesis



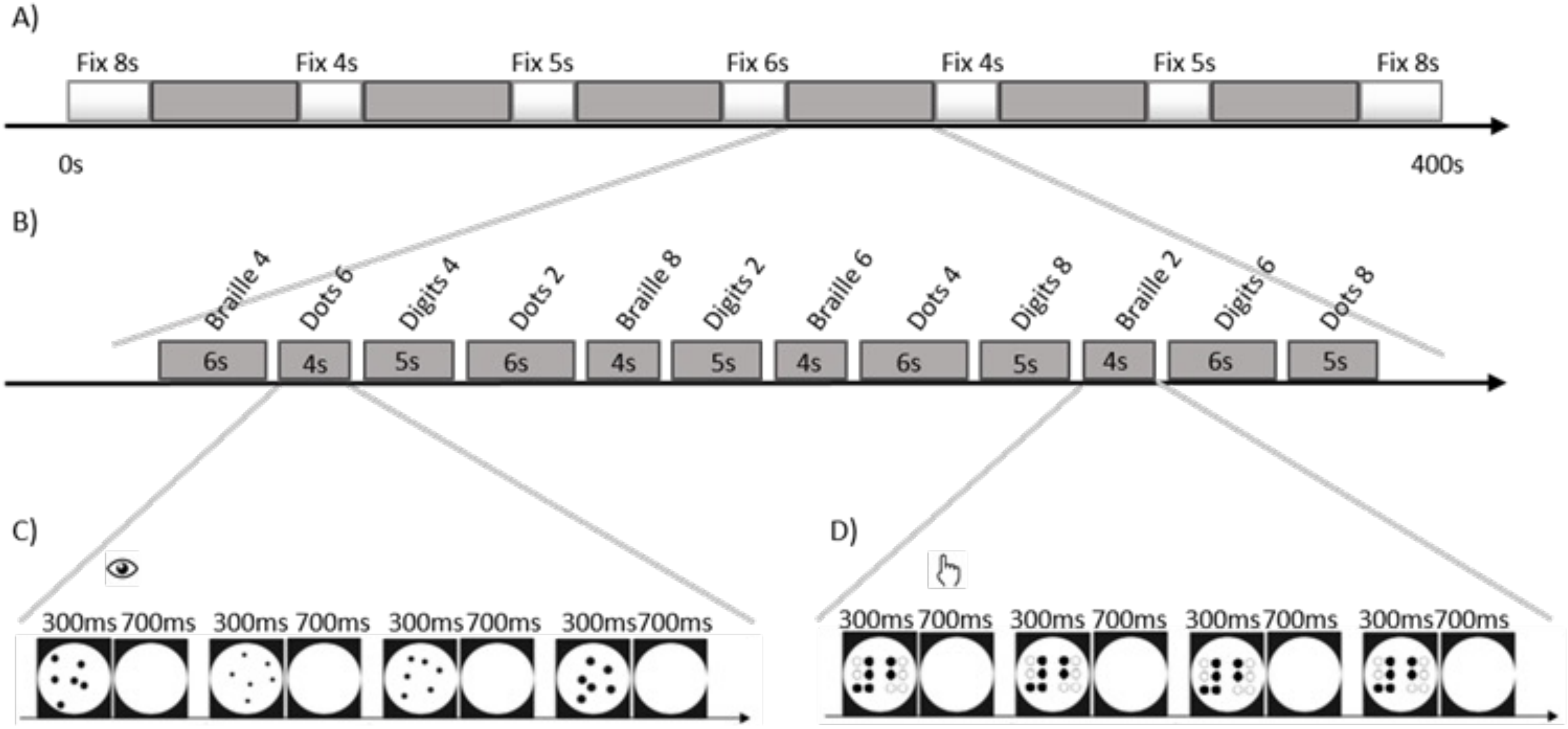
Nieder & Dehaene *Annu Rev Neurosci* 2009

Methods

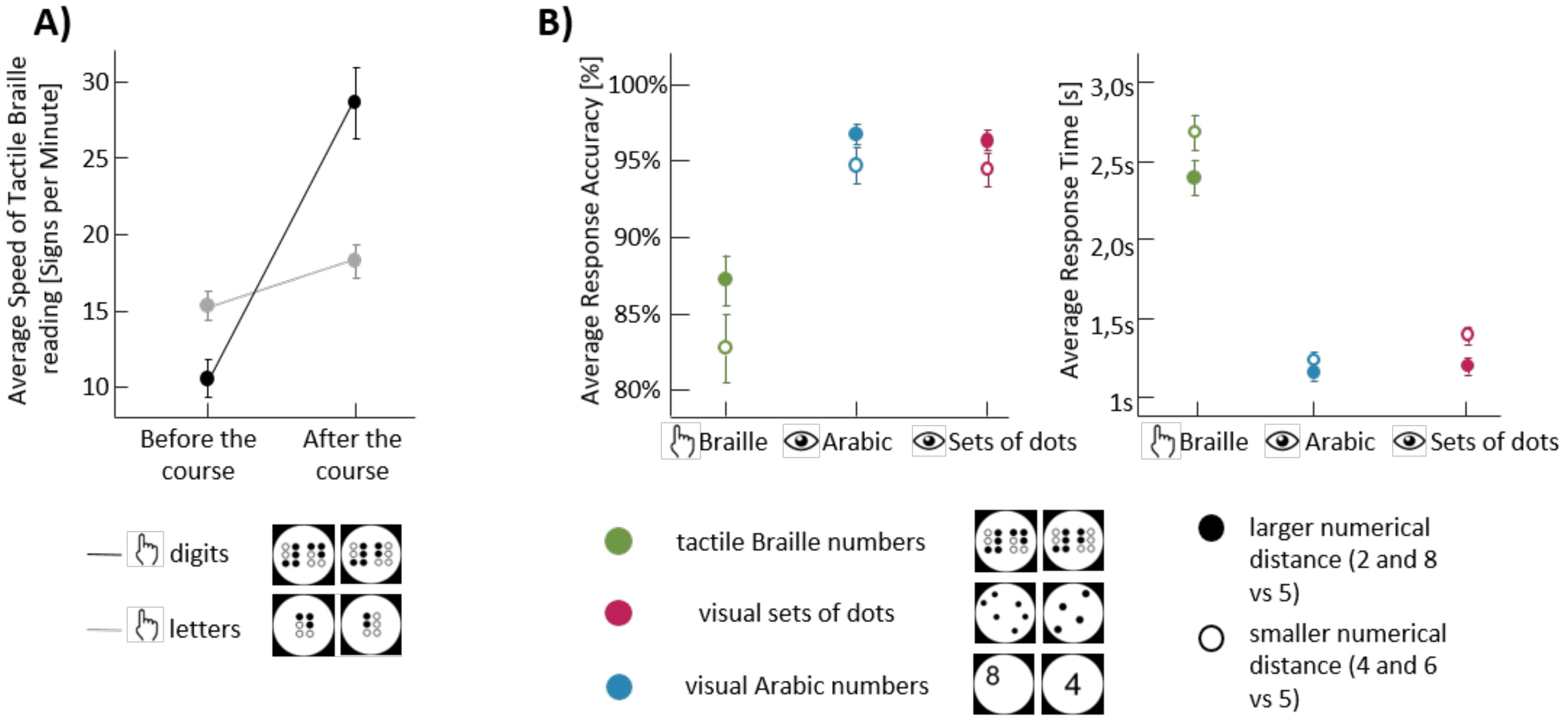


Maria Czarnecka Kasia Raczy

25 healthy adults were trained in reading Braille numbers before scanning



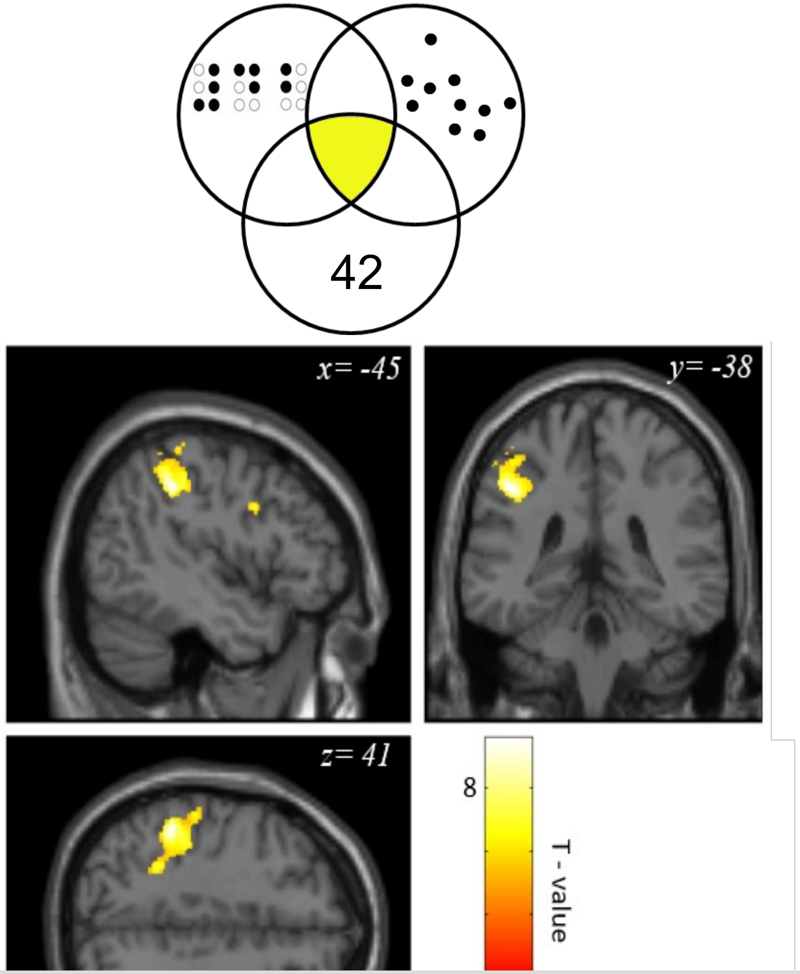
Czarnecka et al., 2023



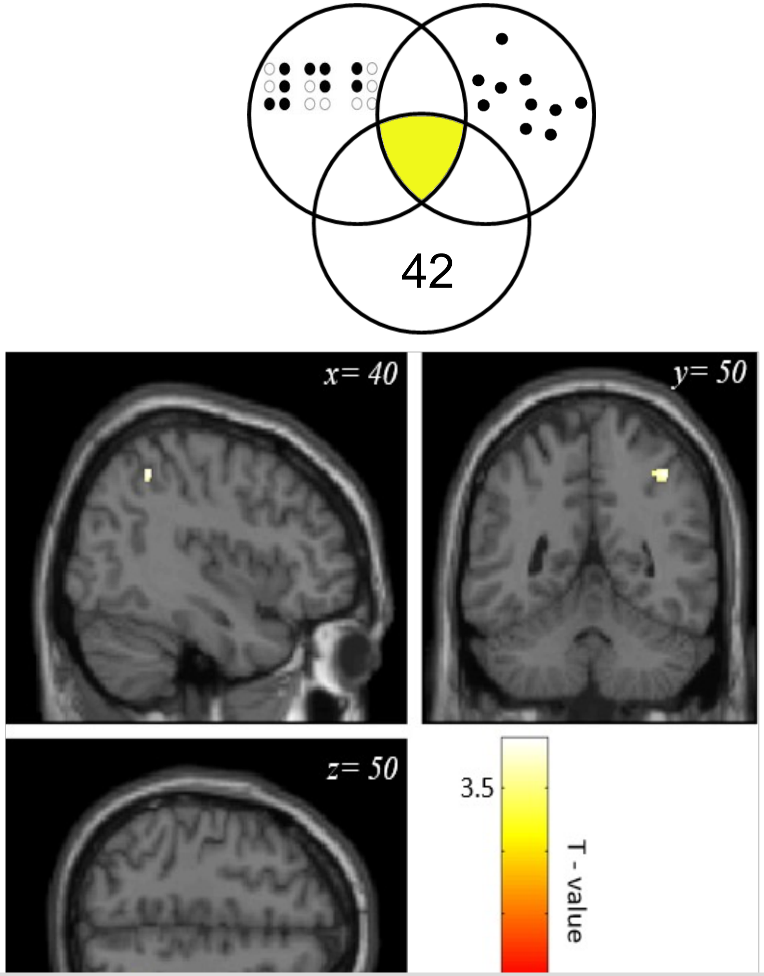
- ➢ Training improved Braille reading
- ➢ Braille reading speed slower than blind Braille readers & slower than visual reading

Czarnecka et al., 2023

Activation



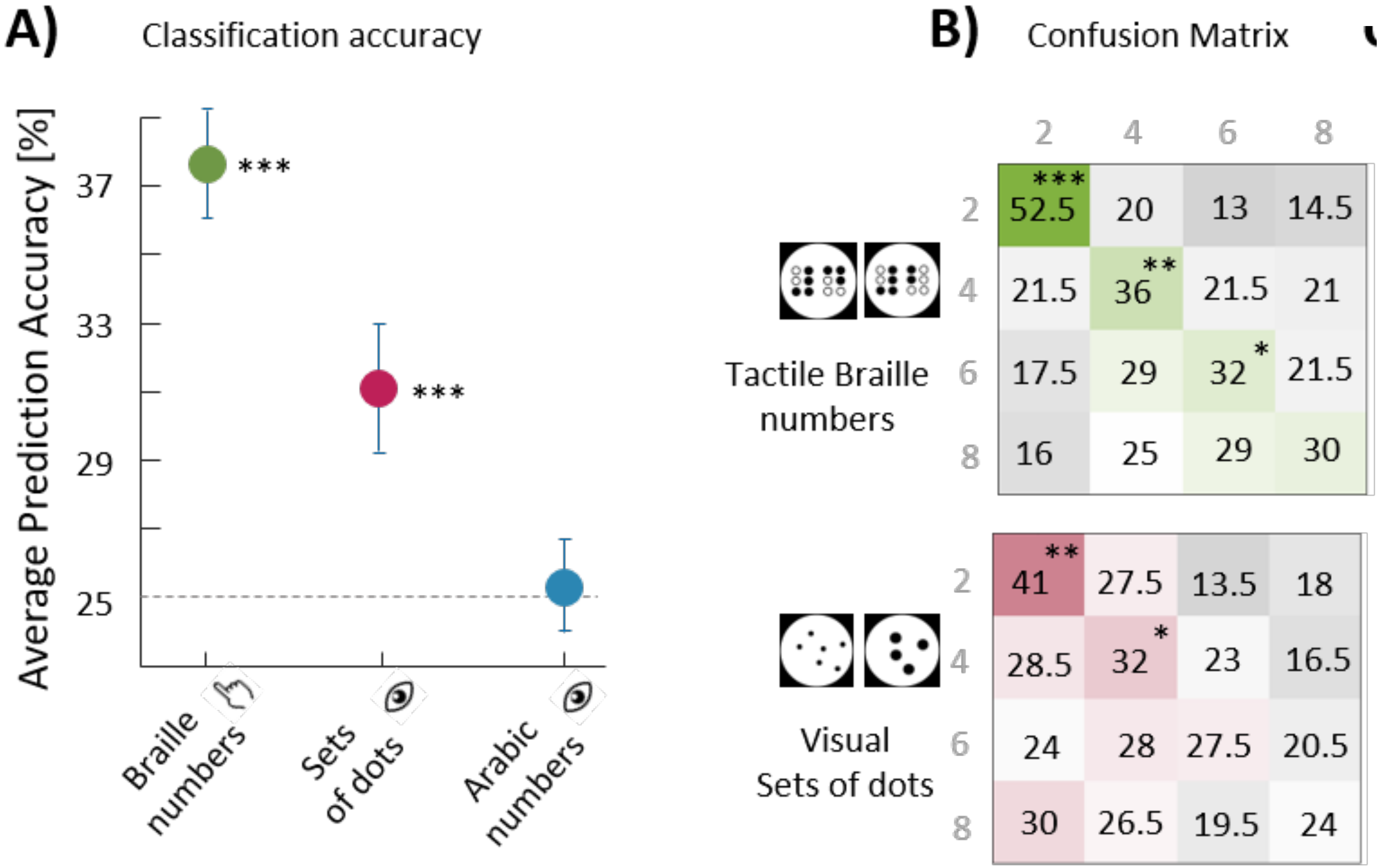
Distance effect



- GLM revealed overlap in IPS

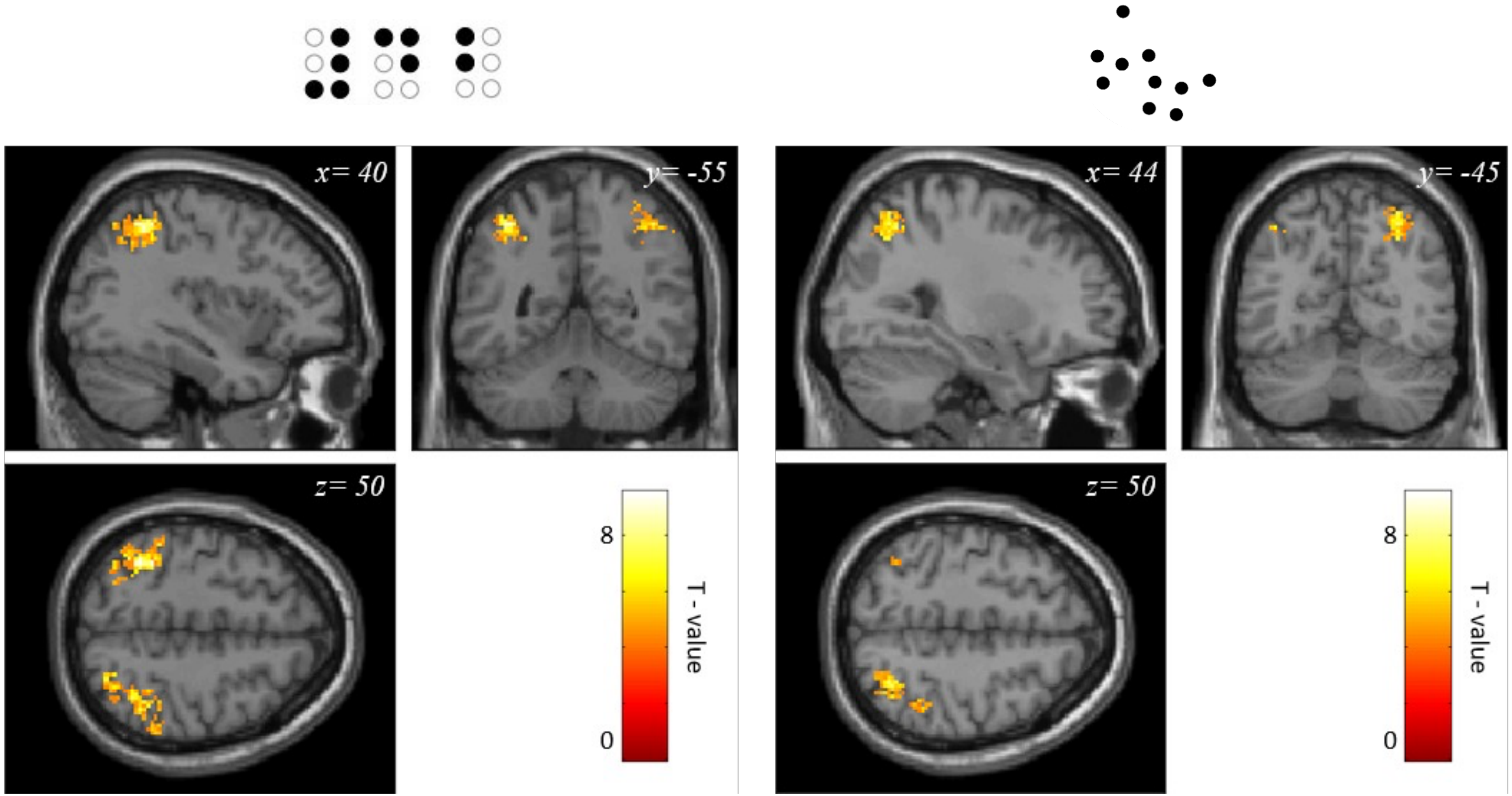
Czarnecka et al., 2023

MVPA in parietal cortex ROI



- ➢ MVPA allowed successful decoding of number information in Braille notation & with sets of dots
- ➢ No above-chance classification for Arabic digits
- ➢ No generalization

Searchlight MVPA



➢ Searchlight MVPA revealed clusters in bilateral IPS that allowed for above-chance classification of number in Braille & sets of dots

Summary

- Overlapping activations in IPS for all three notations.
- Above-chance accuracy classification for Braille & sets of dots but not for Arabic digits.
- Above-chance classification despite symbolic nature of Braille numbers
- Searchlight MVPA converges on IPS for Braille & sets of dots
- No generalization between notations

Czarnecka et al., 2023

Discussion

- IPS key region for processing numerical information
- Overlapping activity & MVPA within-notation classification WITHOUT generalization are at odds with the assumption of a common coding of number information irrespective of notation/modality
- Results support idea of overlapping but distinct neural circuits in IPS for different notations

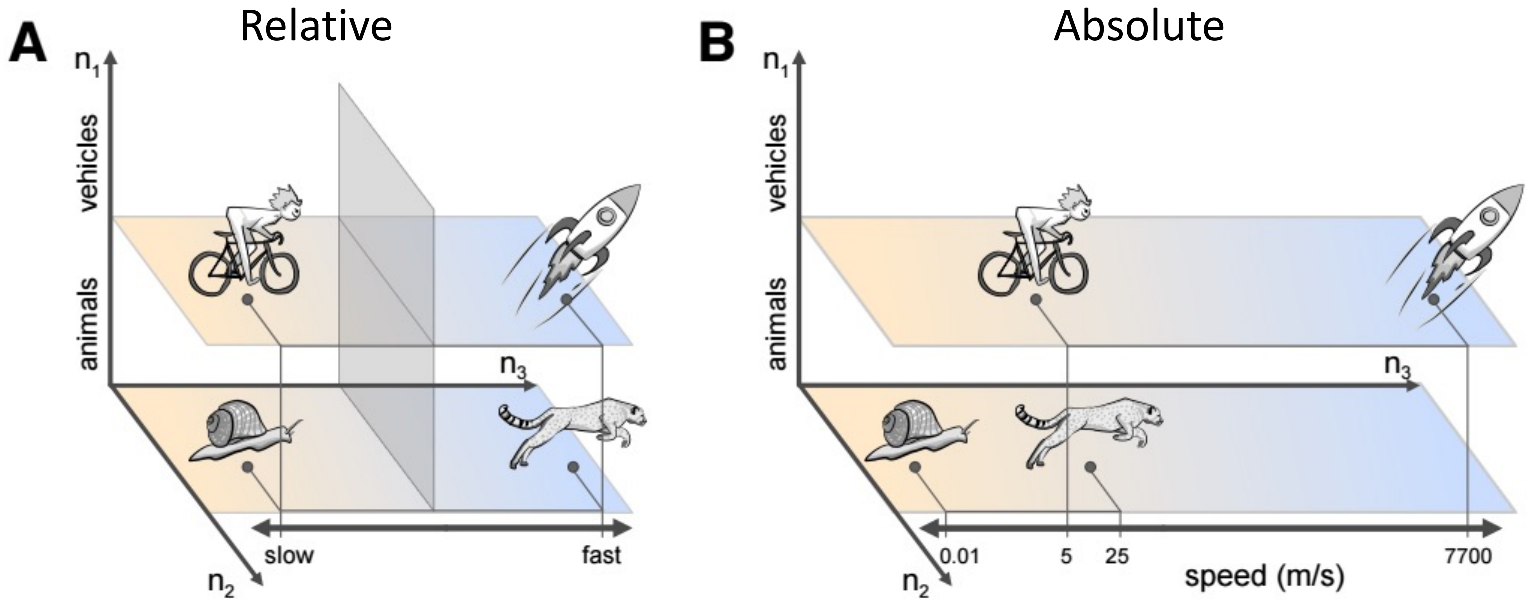
Czarnecka et al., 2023

Numerical Cognition

Neurocognitive **Architecture** of Numerical Information

- Numerical magnitude is specific to
  - Mode (simulatenous / sequential)
  - Modality (visual / tactile)
- Numerical Magnitude is coded
  - Absolute
  - Logarithmically compressed

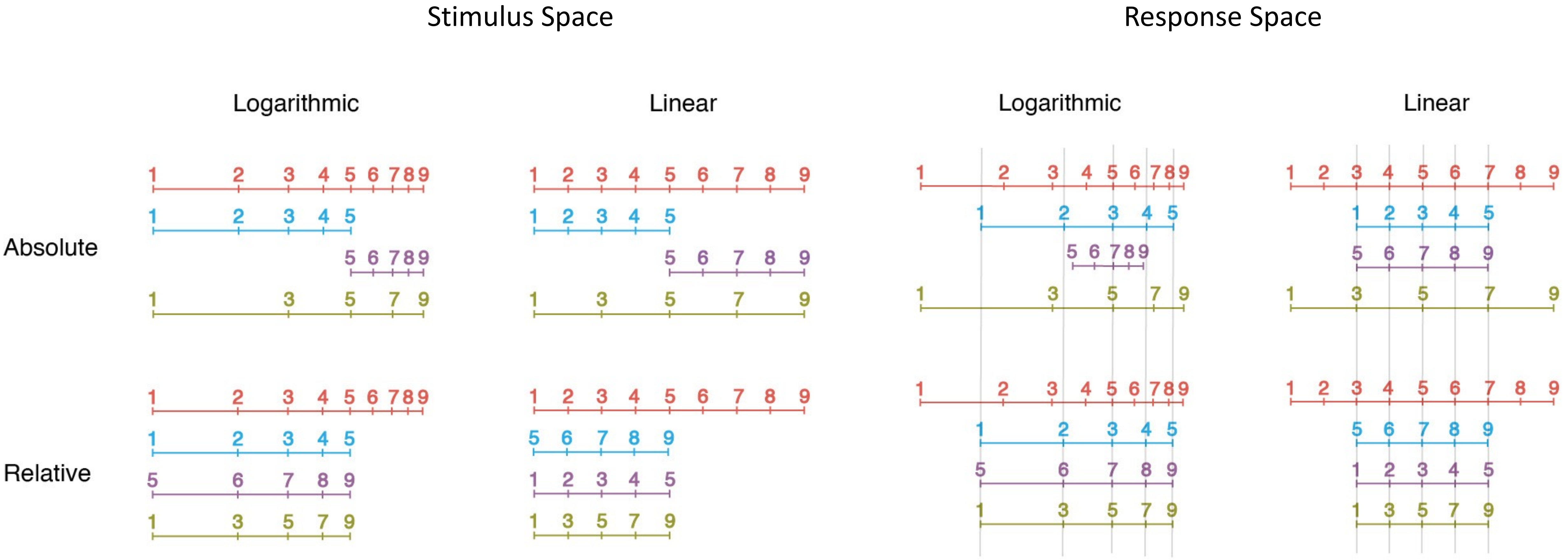
Is numerical magnitde coded in absolute or relative terms?



Sheahan et al., Neuron 2021

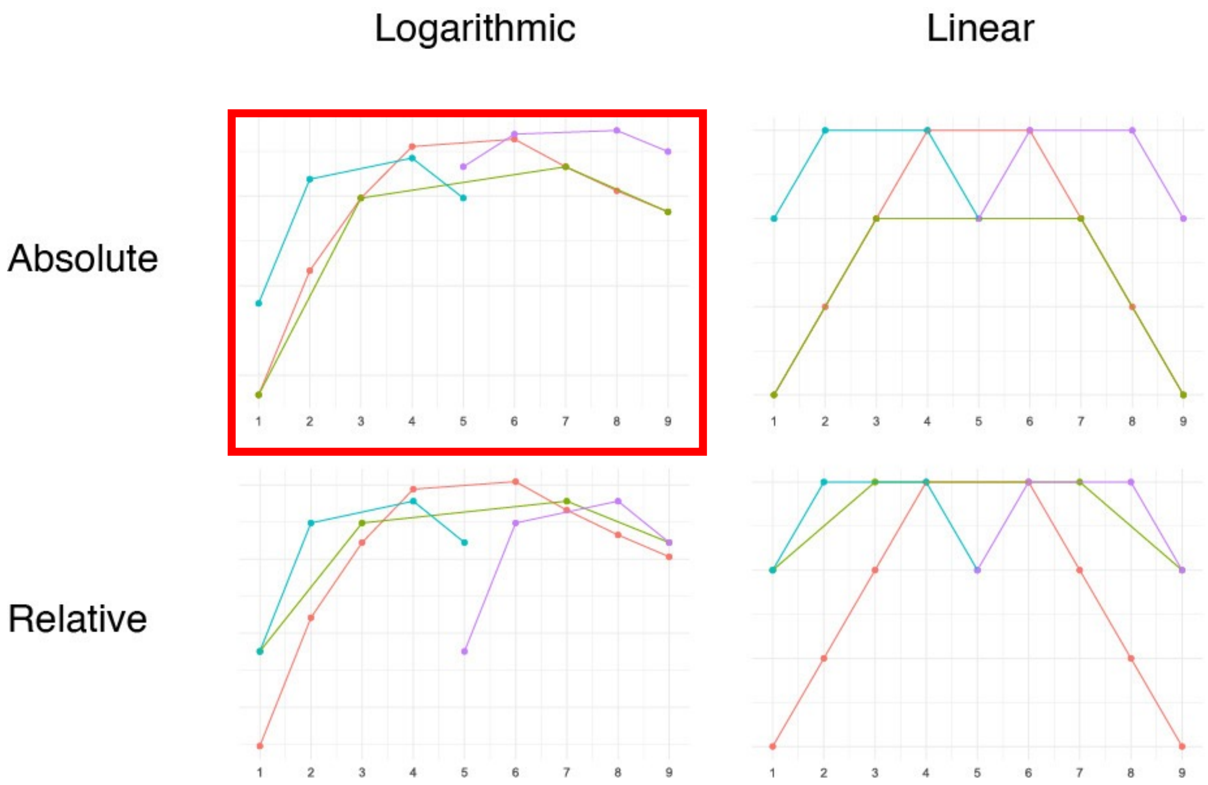
Alexis Garsmeur

How is number coded in the context of a comparison task?



Garsmeur & Knops, in preparation

Predicted reaction time profiles



Hypotheses

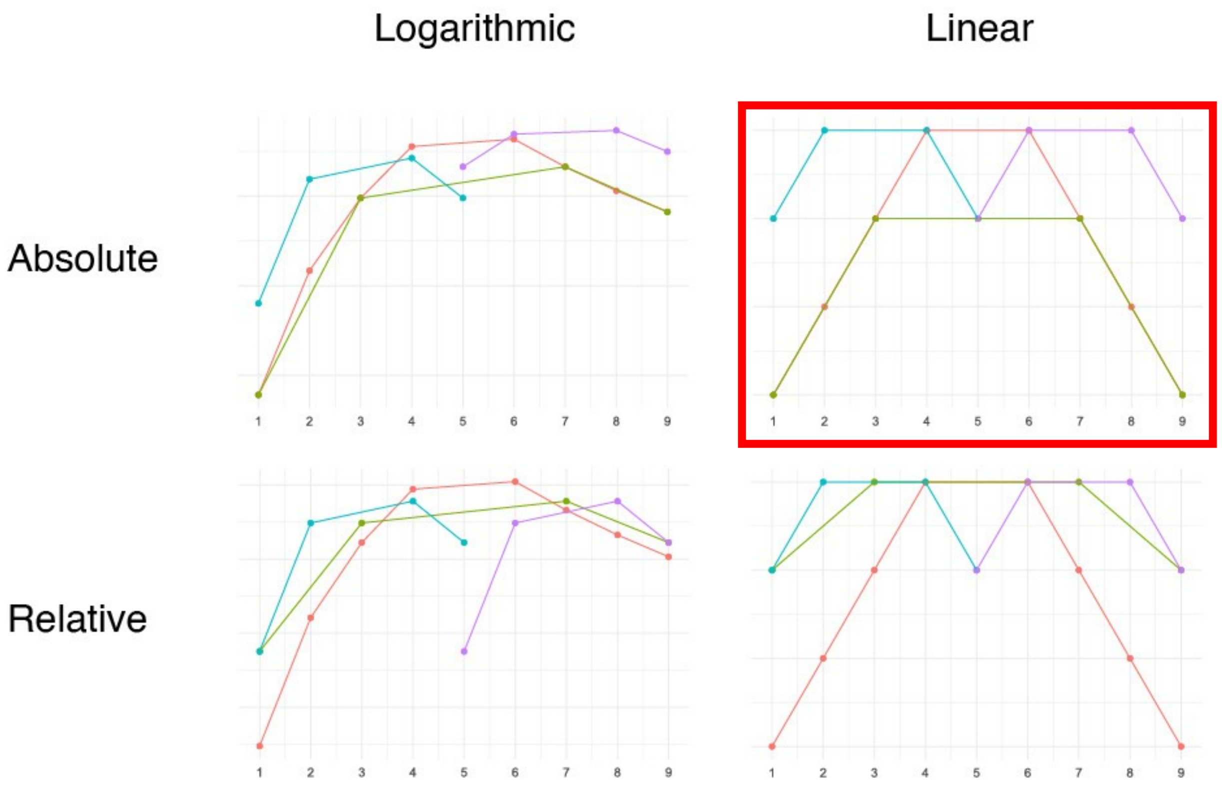
Absolute or relative?
Reaction time and slopes of a regression of absolute magnitude on reaction times should not differ between the large partial and the large complete item set (when only common numbers are included: 1,3,7, and 9).

Log or linear?
Due to the logarithmic compression, we expect that reaction time and slopes of a regression of absolute magnitude against reaction time should **differ** for the number ranges before and after the reference in each set. More specifically, reaction time should be higher for numbers after the reference and the slopes should be shallower.

Reaction times for the small end set are longer than for the small begin set.

Garsmeur & Knops, in preparation

Predicted reaction time profiles



Hypotheses

Absolute or relative
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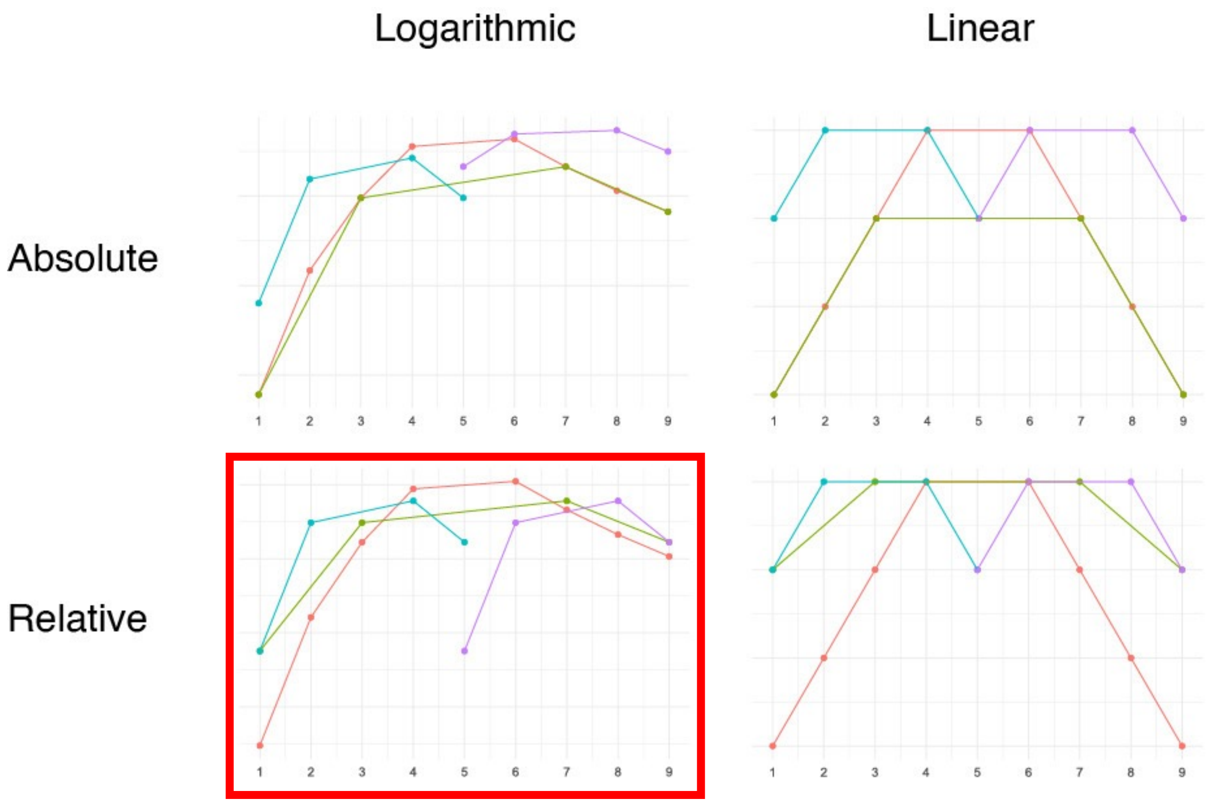
Linear or log?
The slopes and reaction time of a regression of absolute magnitude against reaction time should **not differ** for the number ranges before and after the reference in all sets. (RT patterns should be symmetric with respect to the reference number.)

RTs for the small end set do not differ from the small begin set.

Garsmeur & Knops, in preparation

Predicted reaction time profiles



Hypotheses

Absolute or relative

slopes and reaction time of a regression of the numbers' positions against reaction times should be identical for the small begin, small end and large partial sets.

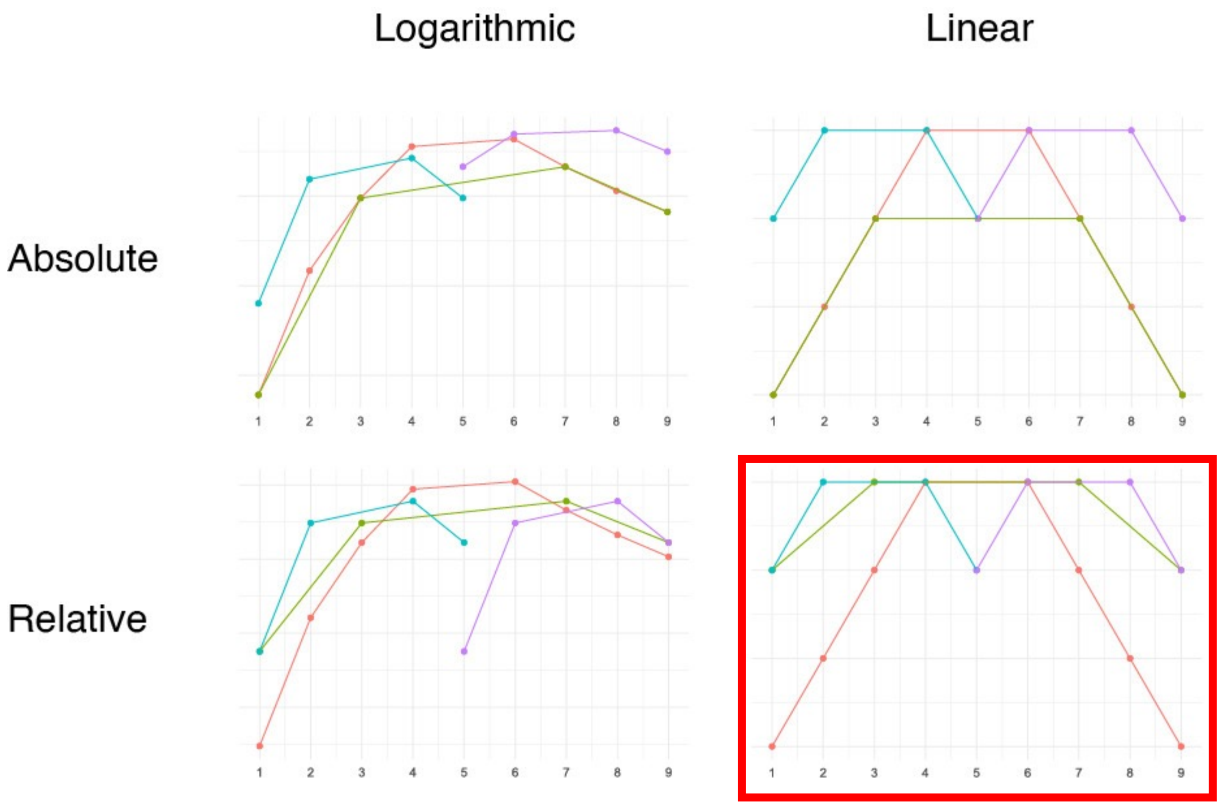
Linear or log?

Slopes and reaction time of a regression of absolute magnitude against reaction time should **differ** for the number ranges before and after the reference in each set. More specifically, reaction time should be higher for numbers after the reference and the slopes should be shallower.

RTs for the small end set are larger than for the small begin set.

Garsmeur & Knops, in preparation

Predicted reaction time profiles



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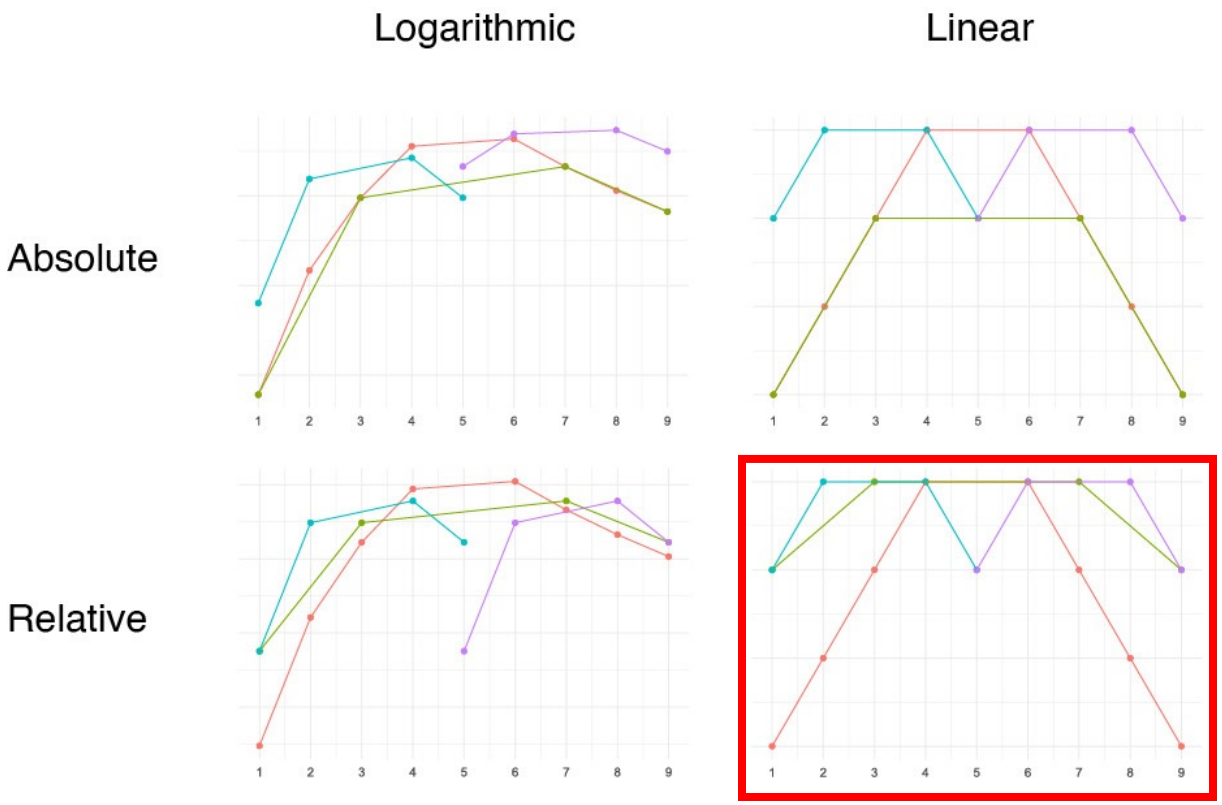
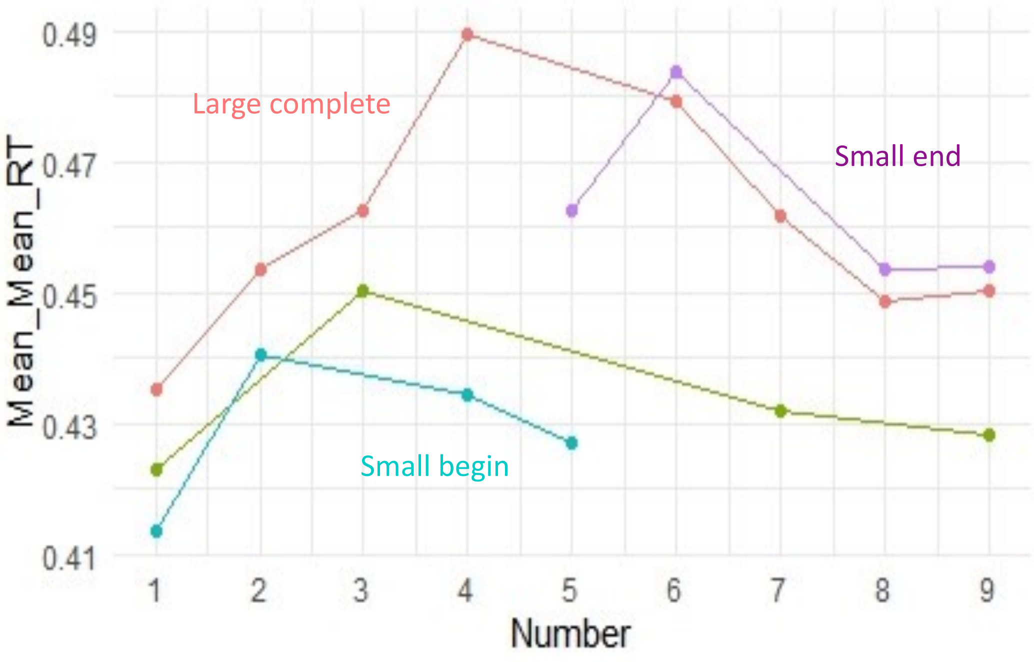
Garsmeur & Knops, in preparation

Predicted reaction time profiles



Results



Garsmeur & Knops, in preparation

None of the predicted profiles totally matches the results.

We find some evidence for **against absolute coding**:
Large partial ≠ large complete

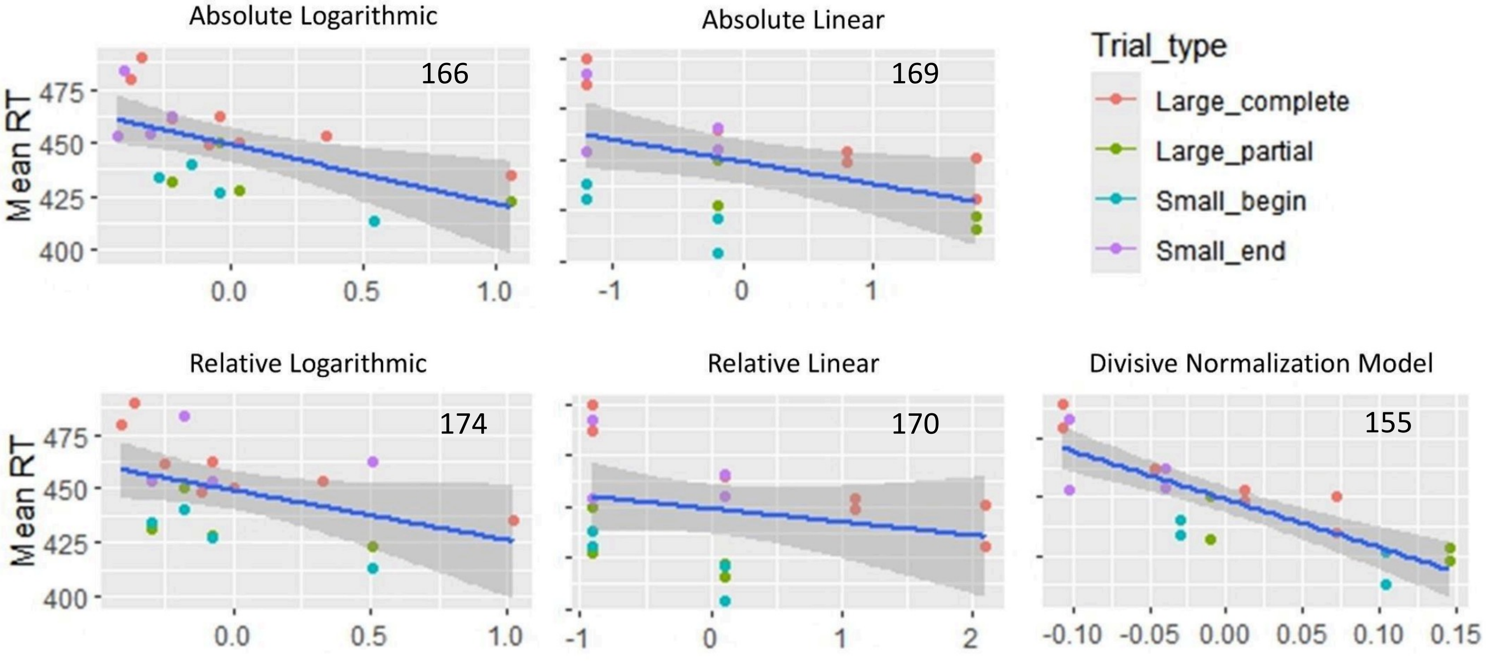
We find evidence **against relative coding**:
Small begin ≠ small end ≠ large partial

We find evidence **against linear** but **supporting logarithmic** coding:
Slopes before reference ≠ slopes after reference
RTs small end > small begin

Behavioral results show that the coding scheme that underlies symbolic numerical magnitude in a comparison task carries signatures of both relative and absolute magnitude coding. While we still need to find a model that captures this behavior, it shows that the MNL is not sufficient as a model.

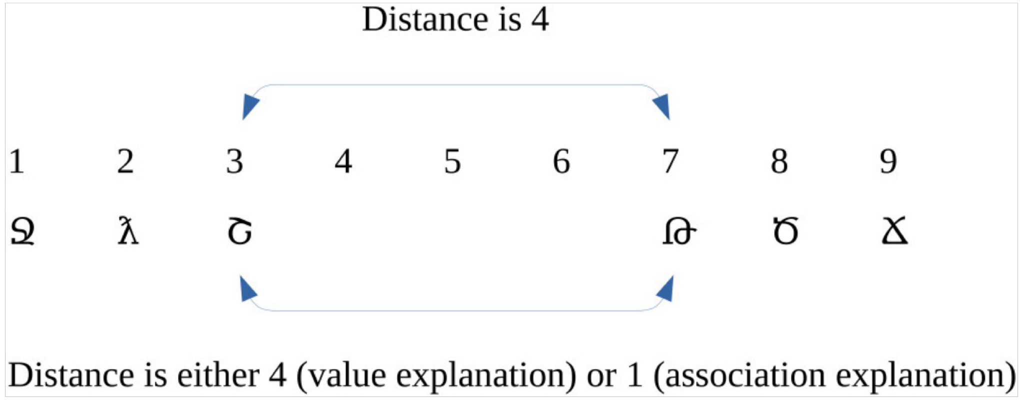
Divisive normalization

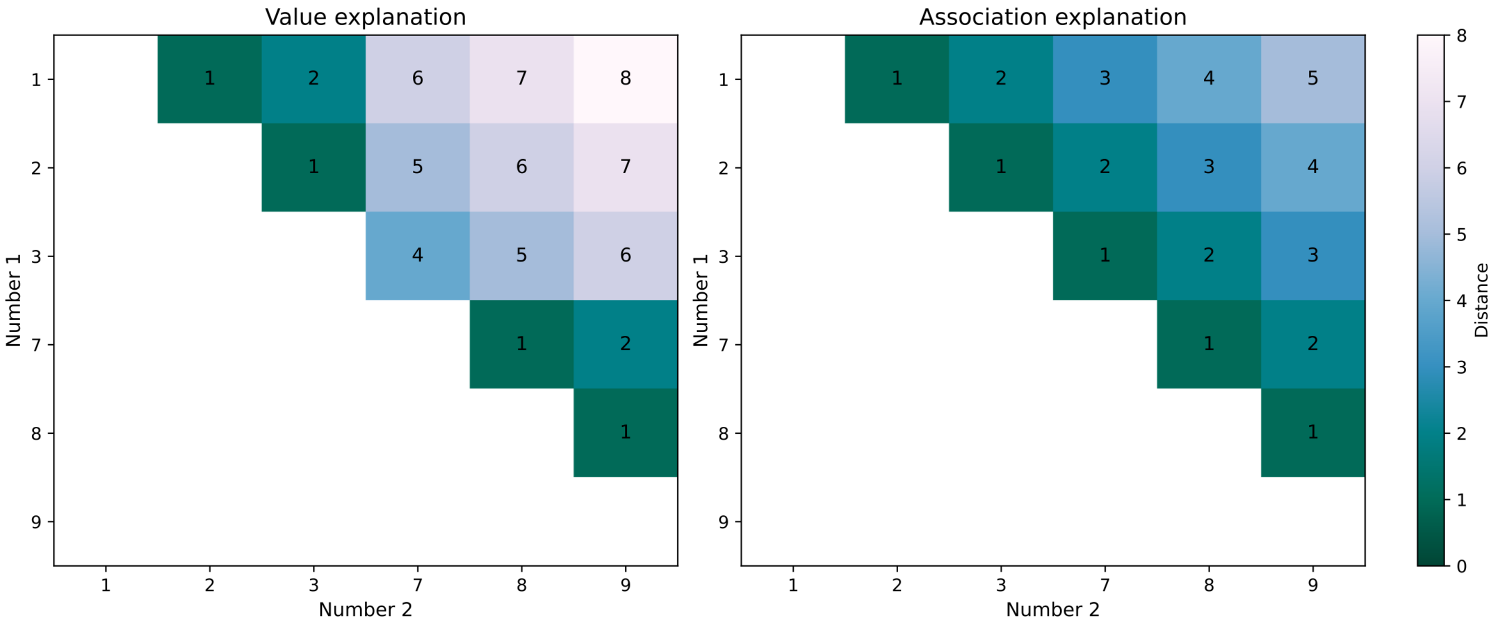
To accomodate limited neural computational power and space for specialized neurons, different physical input dimensions are subject to divisive normalization mechanisms in the cortex. It adapts a given neuron‘s activity to the range (mean absolute magnitude) and spread (the contrast max to min) of the input from a given group of neurons.



Divisive normalization provides the best model (smallest AIC).
Neural mechanisms that are applied to sensory input (and non-symbolic numerosity) may also be used to code for symbolic numbers.

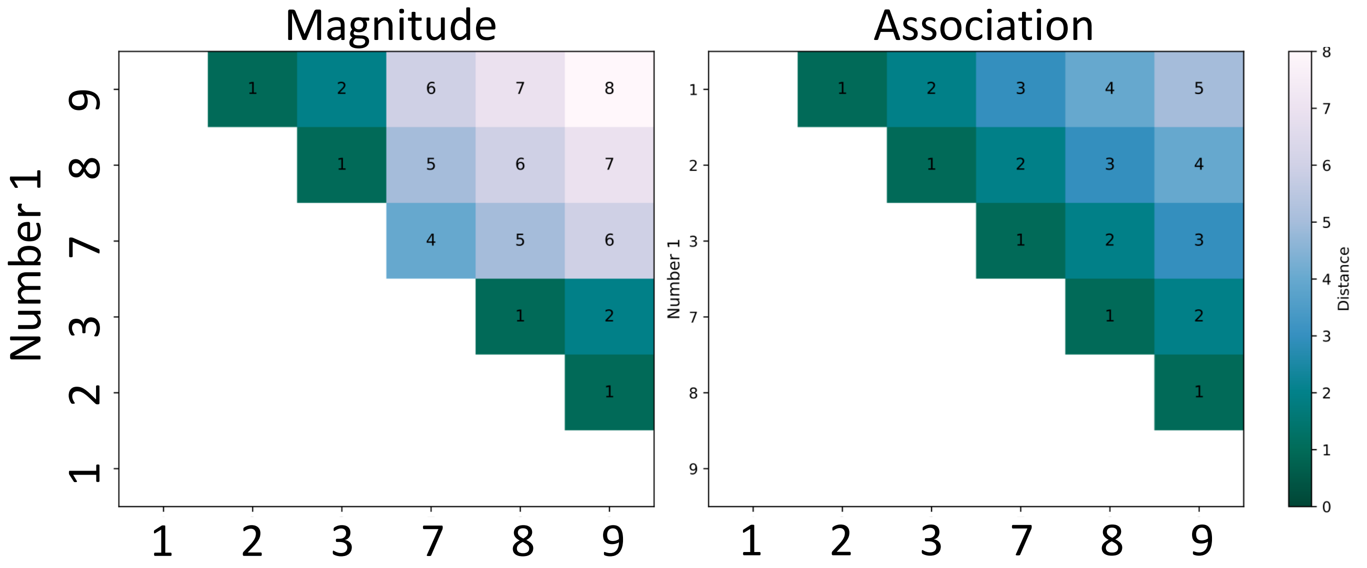
Anastasiia Shivarova



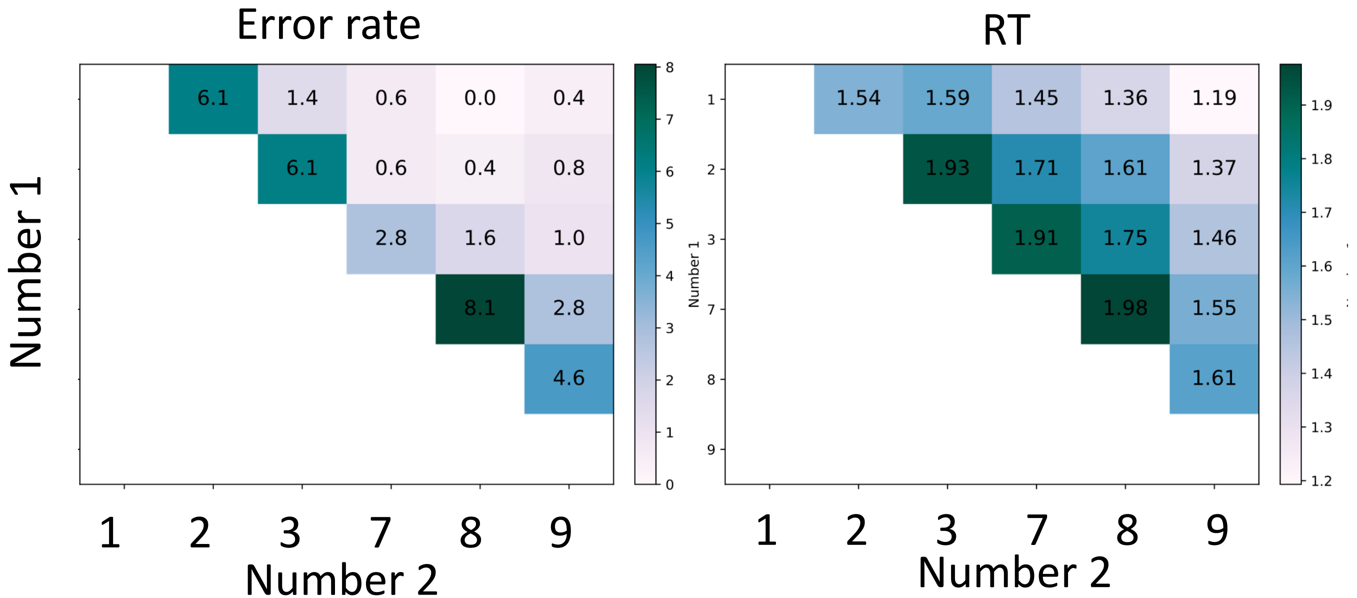


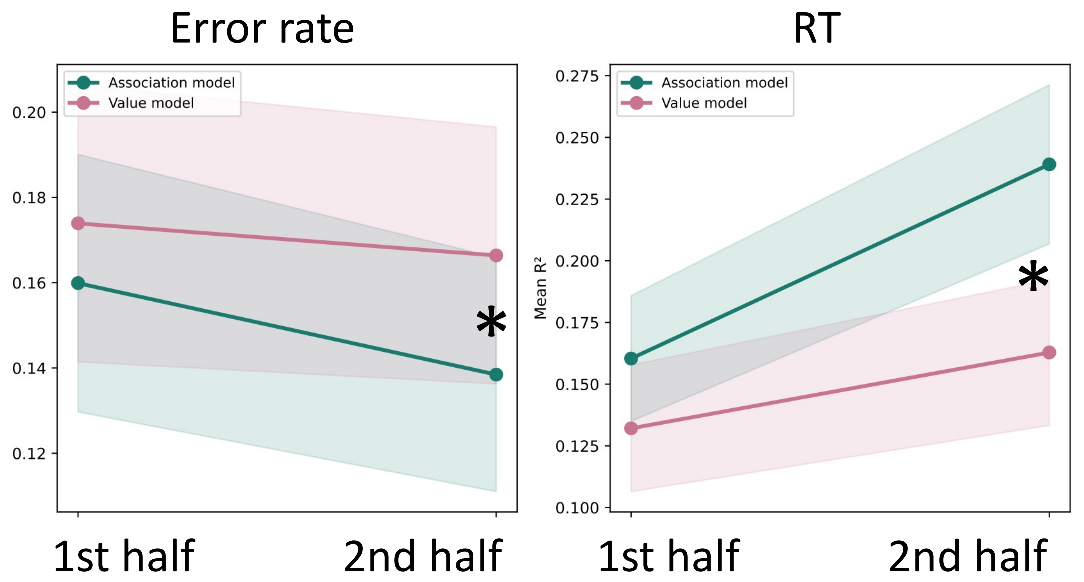
Shivarova & Knops, in preparation

We created 9 symbols, 1 per digit that 25 participants were trained with.
In the comparison task, however, only digits 1,2,3, …, 7,8,9 appeared in a magnitude comparison task.



The association model provides a better description of the data. Advantage increases over time on task, underlining the dynamic creation of a task set – decoupled from absolute magnitude.



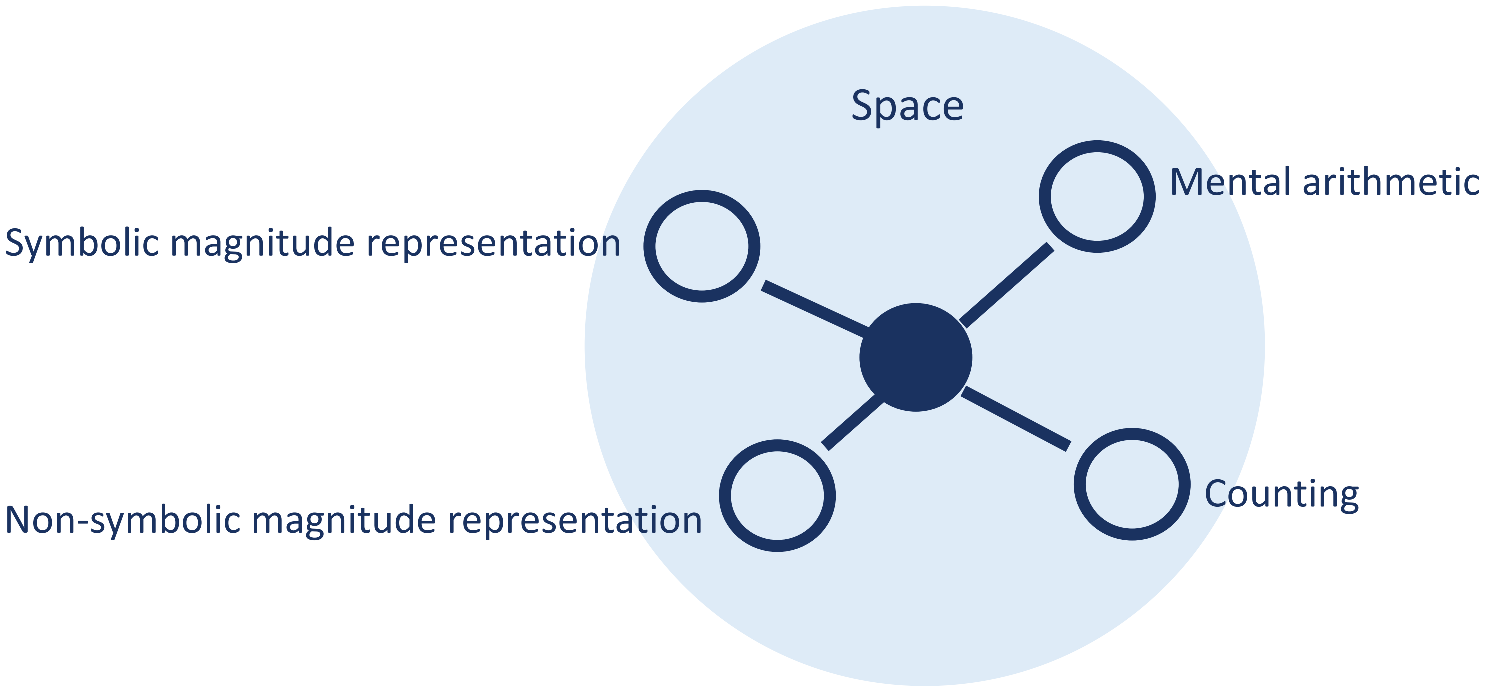


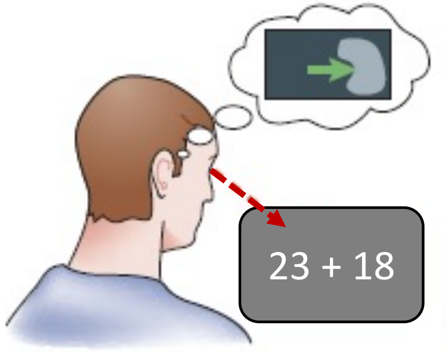
Numerical Cognition

Neurocognitive **Architecture** of Numerical Information

- Numerical magnitude is specific to
  - Mode (simulatenous / sequential)
  - Modality (visual / tactile)
- Numerical Magnitude is coded
  - In a standardized manner (divisive normalization)
  - In task-specific sets

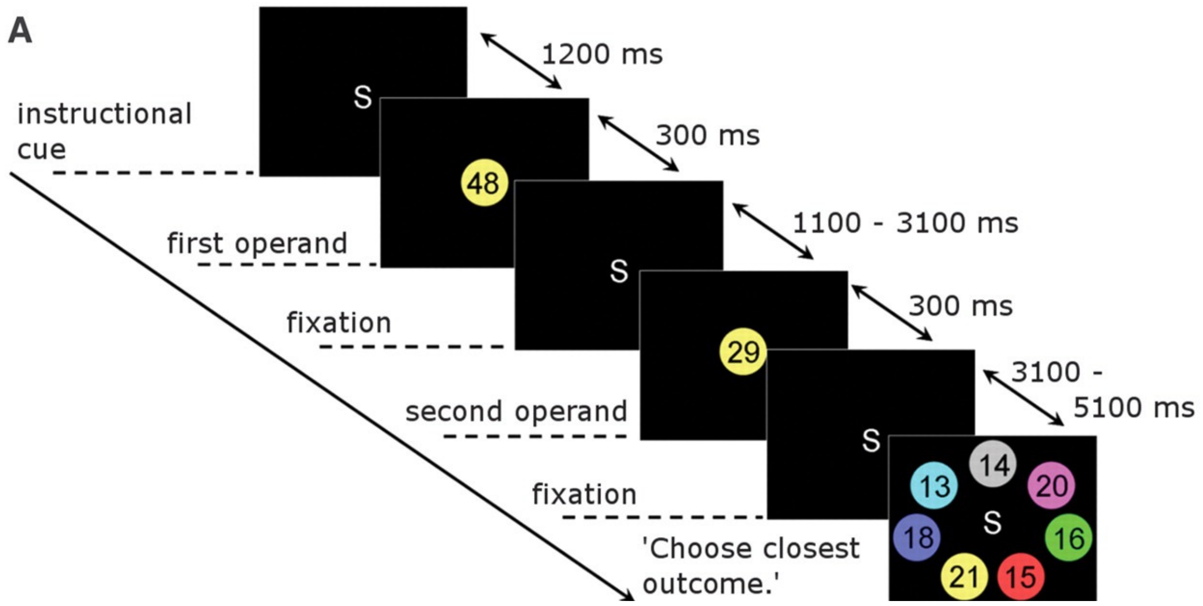
Numerical cognition has many facets





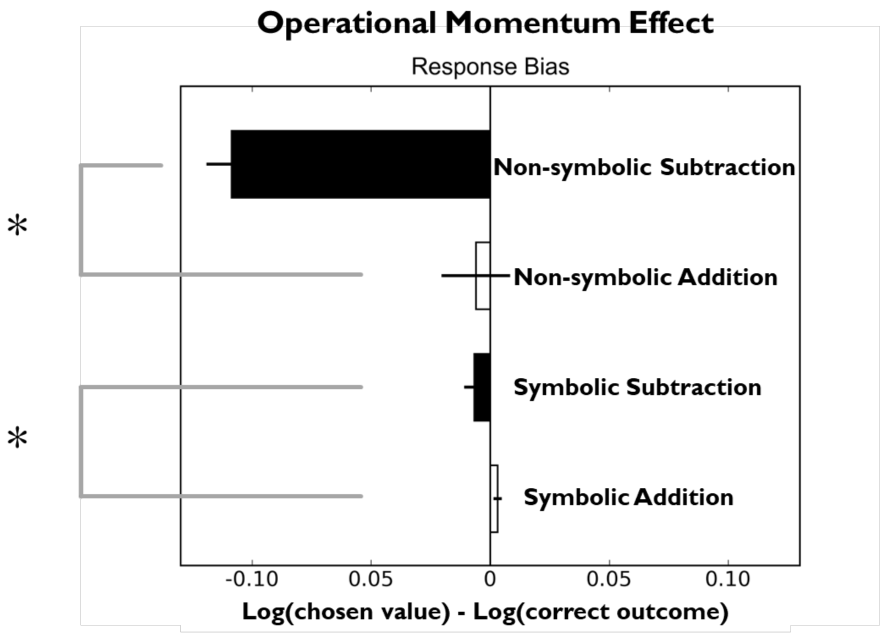
Spatial processes contribute to these functions

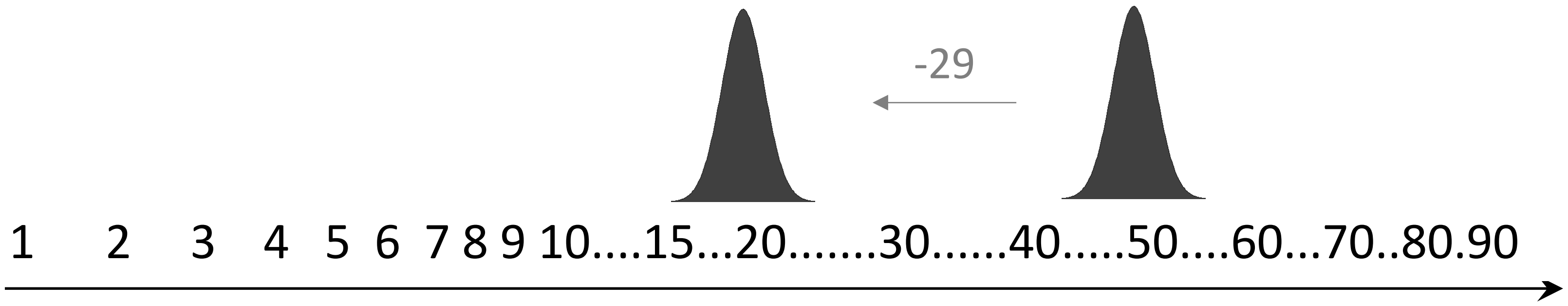
Paradigm



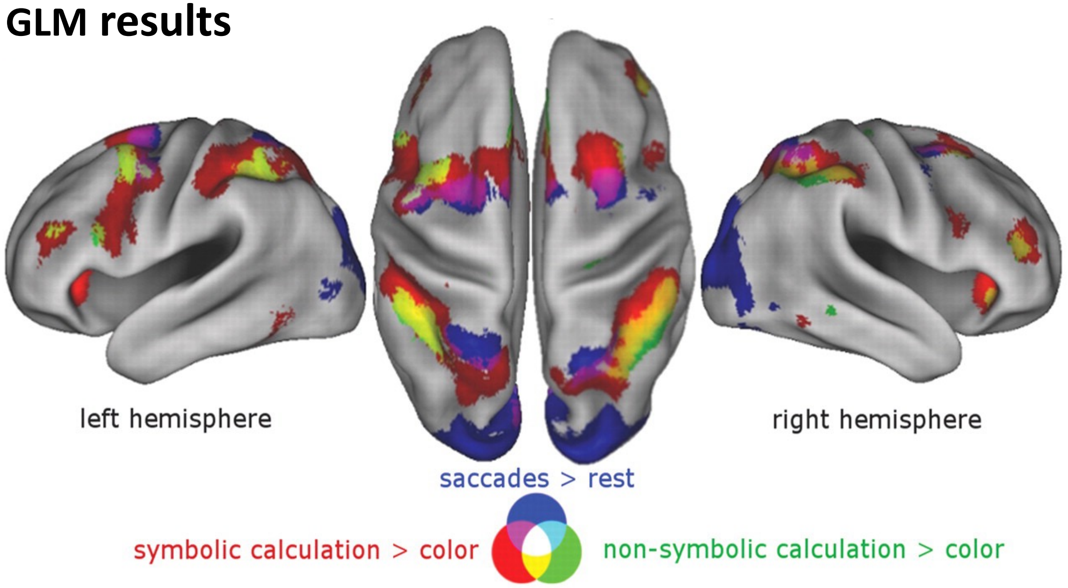
Knops et al. (*Attention Perception & Psychophysics* 2009)

Behavioral results



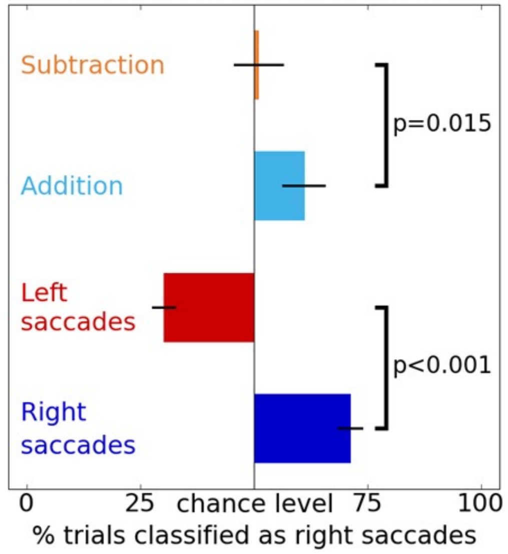


GLM results

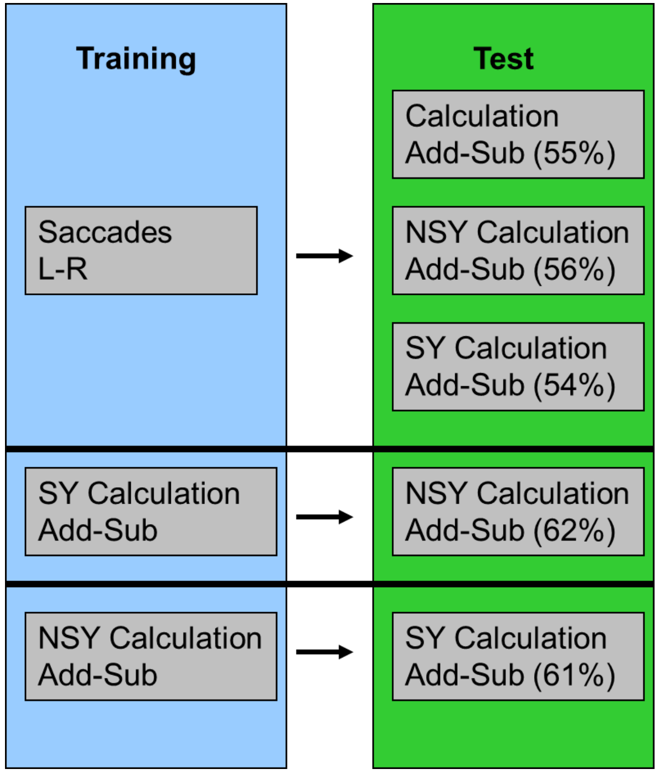


Knops et al. (*Science* 2009)

Decoding results

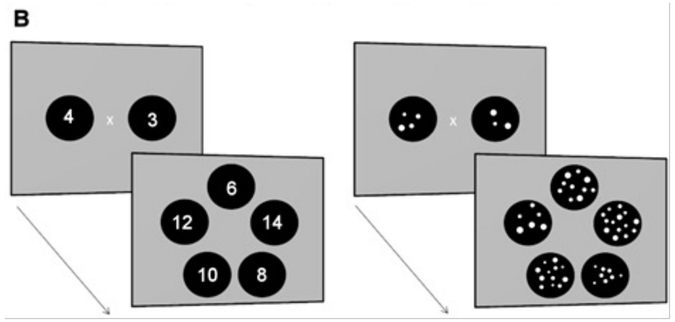


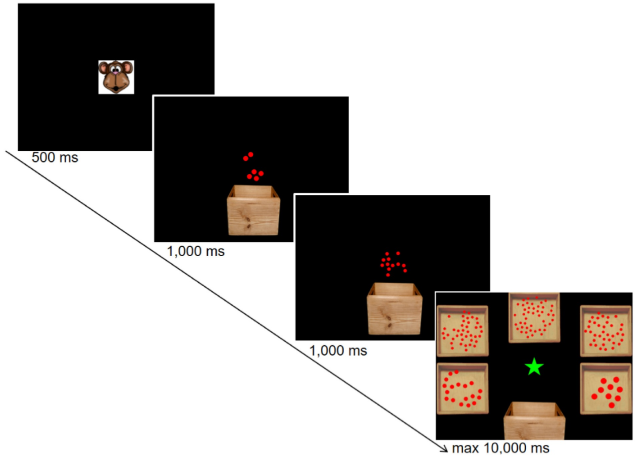
Control analyses



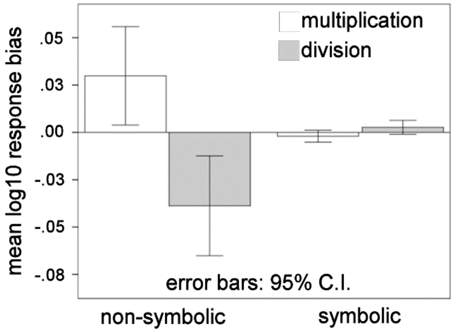
Approximate calculation recruits spatial attention, supposedly because it operates on a spatial representation of numerical magnitude.

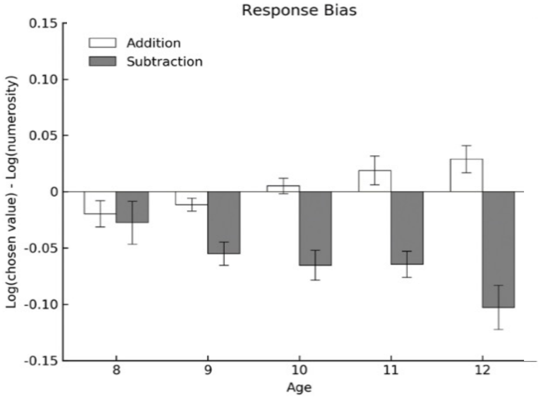
Paradigm





Behavioral results



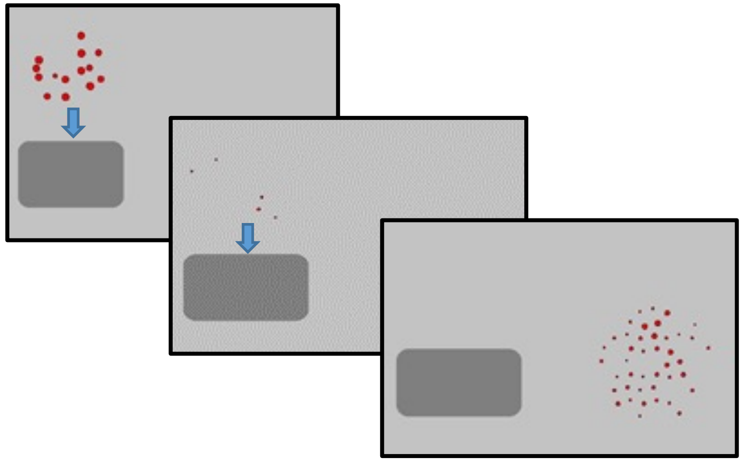


OM effect only present in approximate arithmetic (vs. rote fact knowledge).

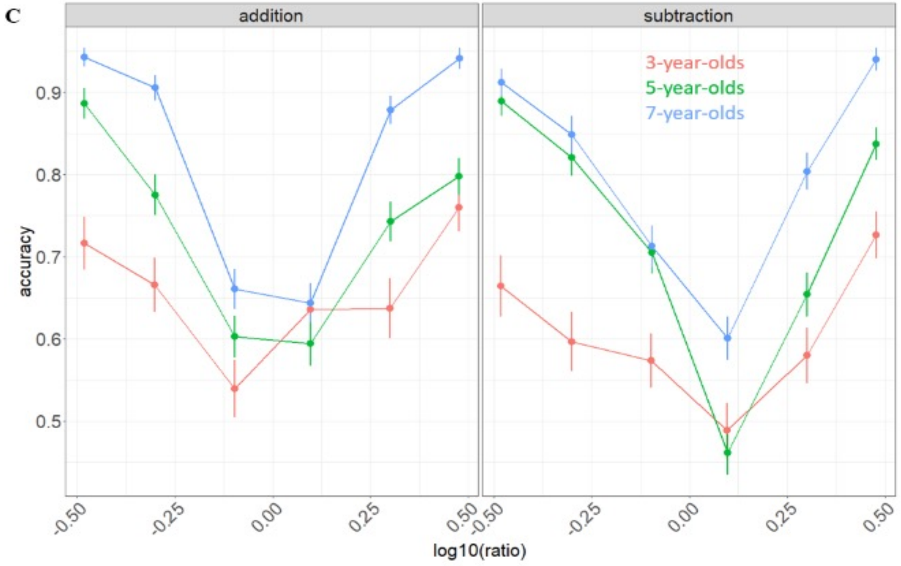
Katz & Knops (PLoS One 2014)

OM effect emerges after ~9 years of age.

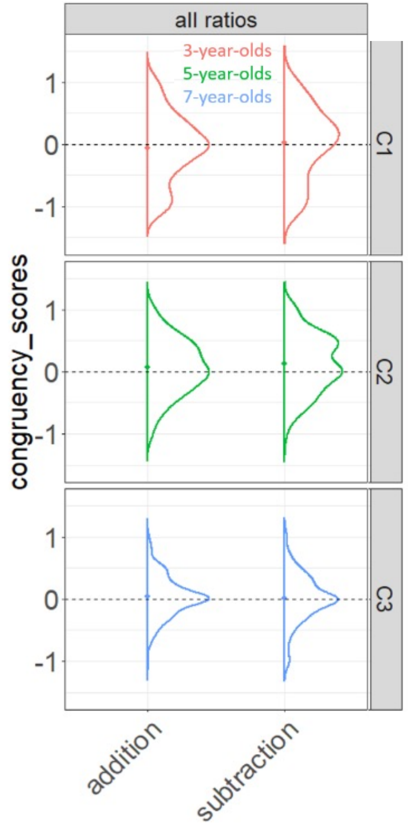
Pinheiro-Chagas et al. (Frontiers 2018)



Accuracy per cohort, operation, ratio

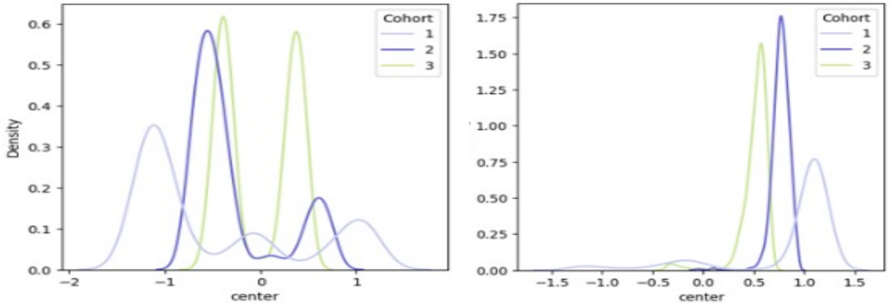


Congruency Score

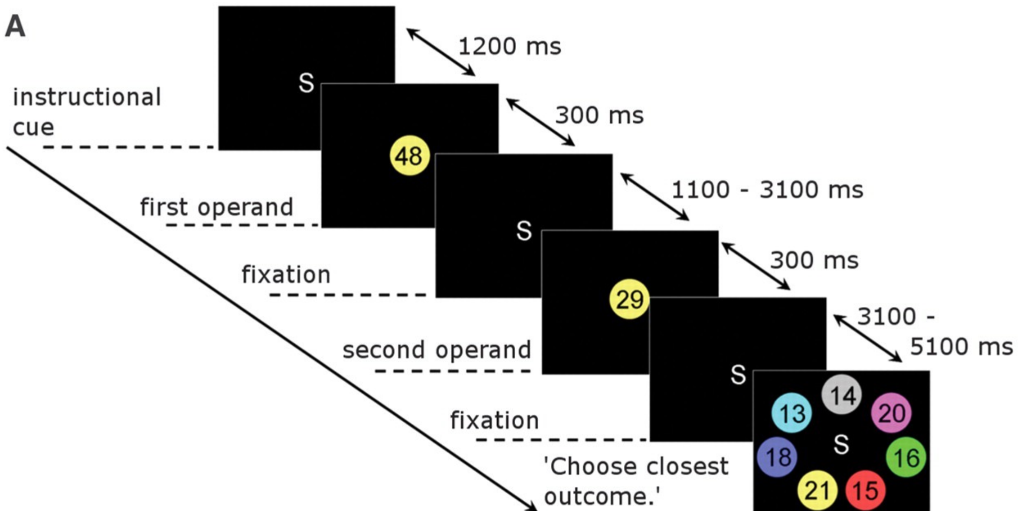


No evidence for an OME in 3- to 7-year-olds

Center parameter of Gaussian fit



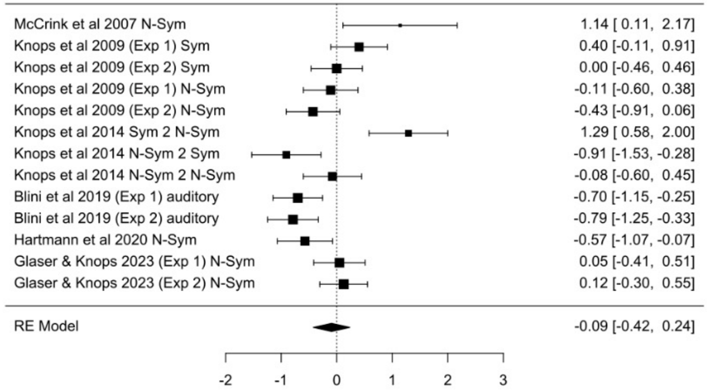
Sixtine Omont-Lescieux



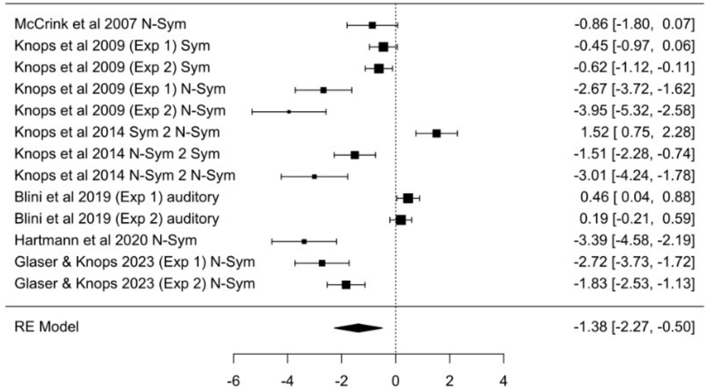
	Addition	Subtraction	Addition - Subtraction
Numerical	✗	✓	✓
Pointing			
Cueing			

Prado & Knops (*Psych Bull Rev*, 2024)

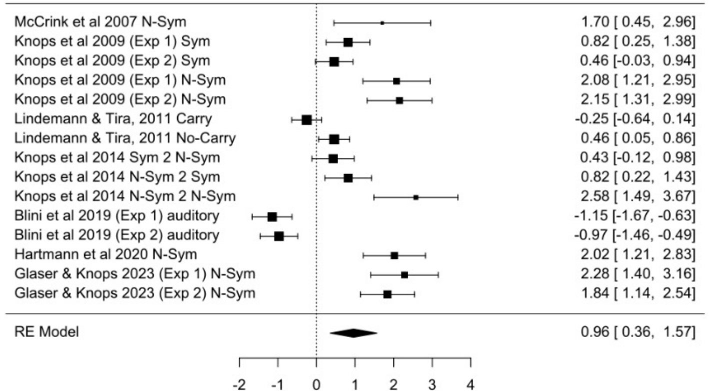
Addition

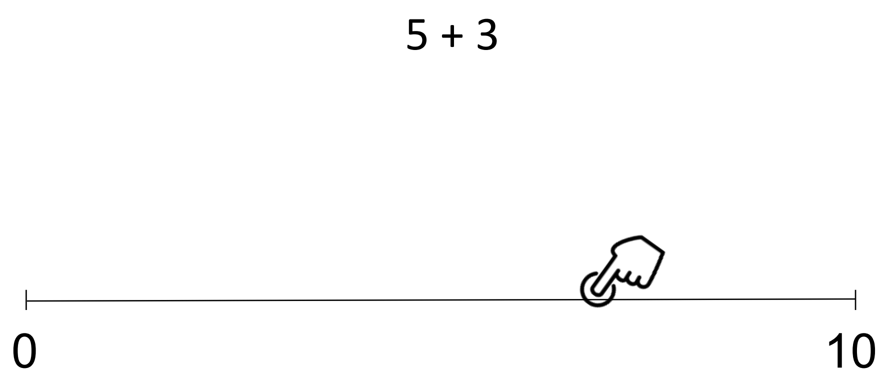


Subtraction



Addition – Subtraction





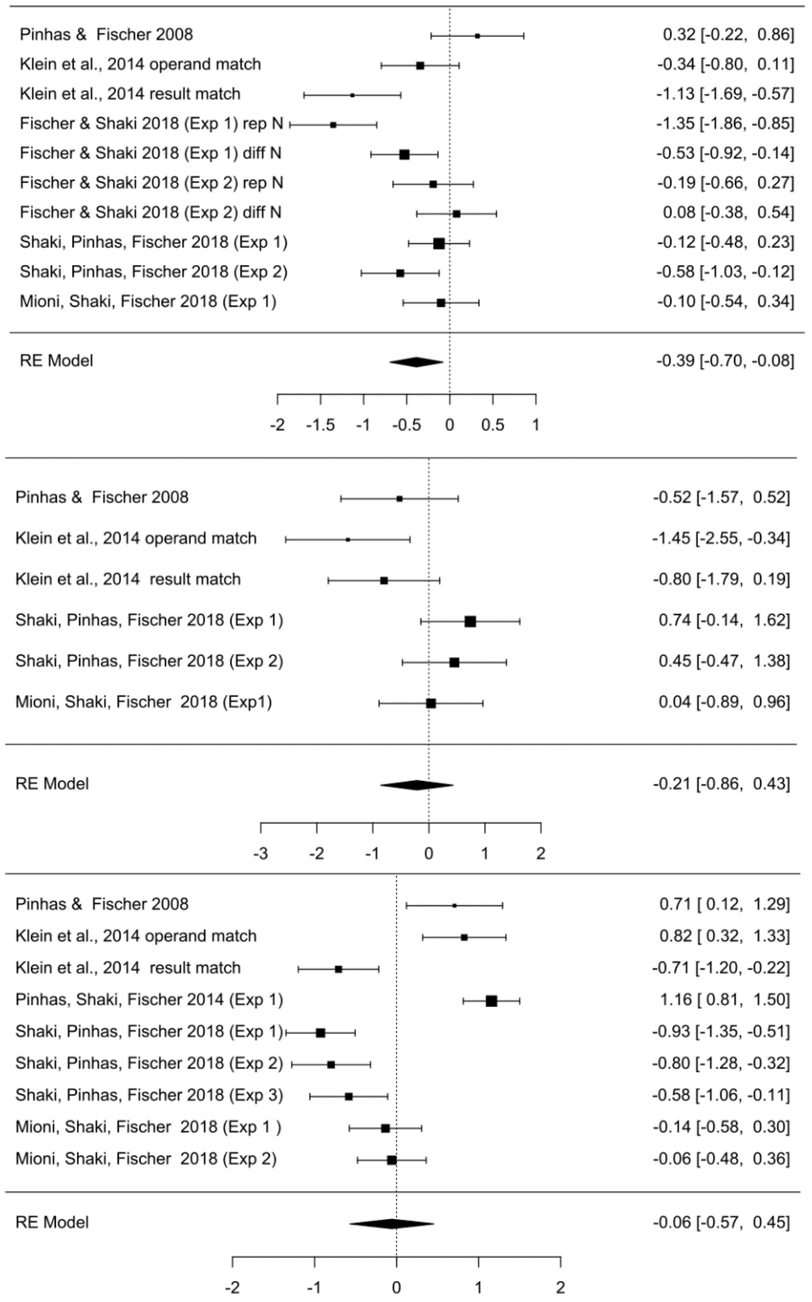
	Addition	Subtraction	Addition - Subtraction
Numerical	✗	✓	✓
Pointing	✓	✗	✗
Cueing			

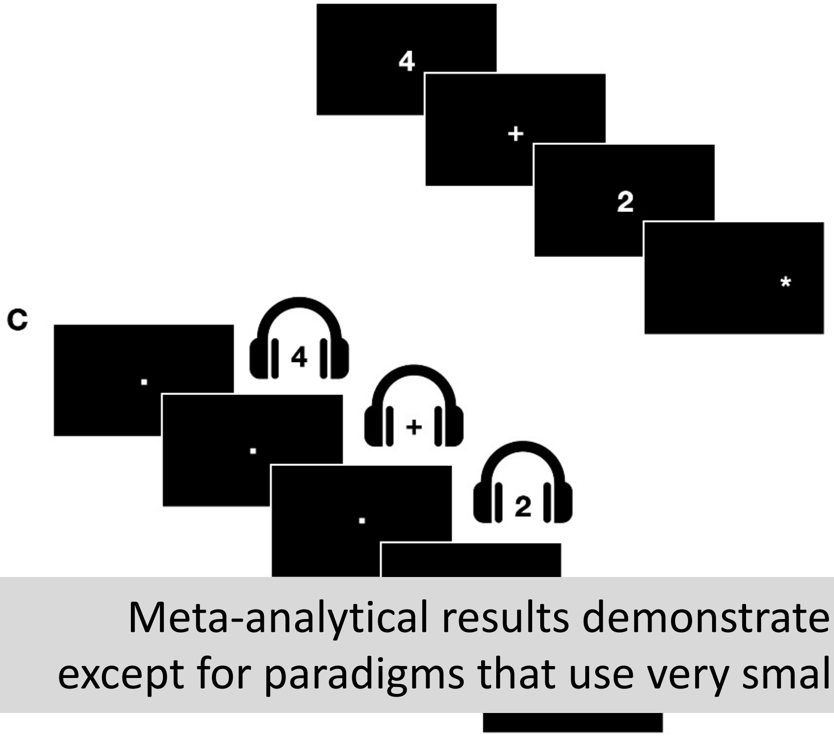
Prado & Knops (*Psych Bull Rev*, 2024)

Addition

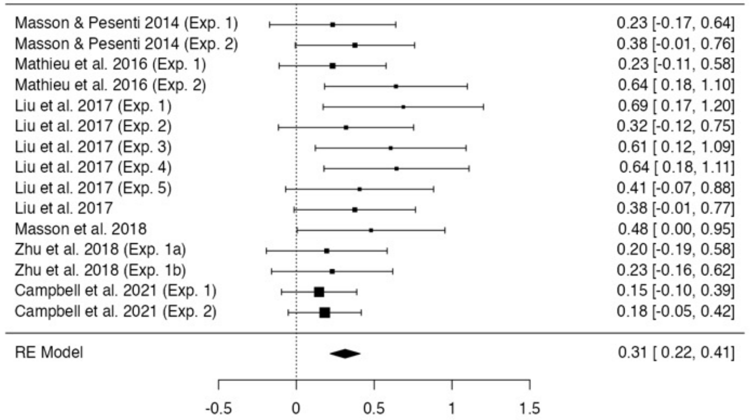
Subtraction

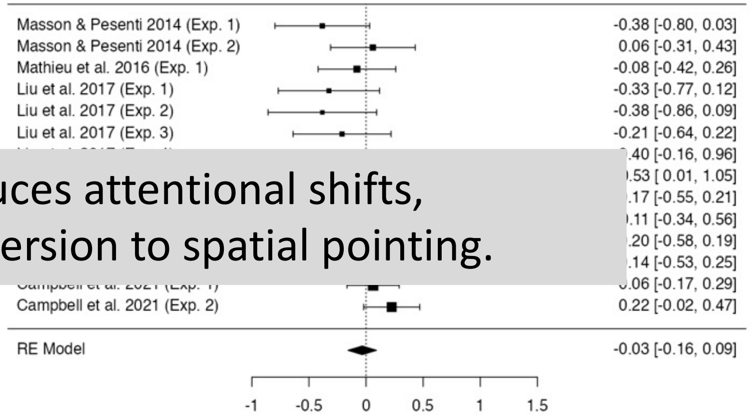
Addition – Subtraction





Addition

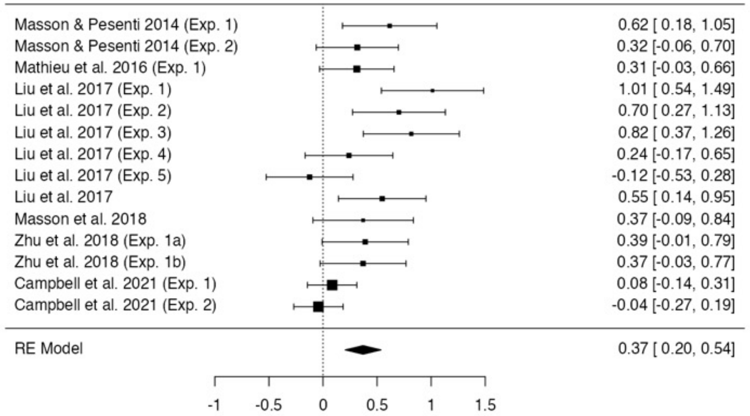




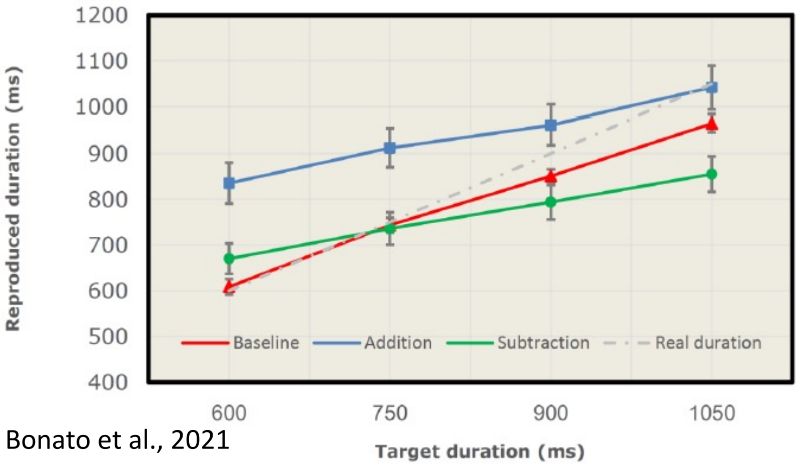
Meta-analytical results demonstrate that mental arithmetic reliably induces attentional shifts,
except for paradigms that use very small problems (<10) and require a conversion to spatial pointing.

	Addition	Subtraction	Addition - Subtraction
Numerical	✗	✓	✓
Pointing	✓	✗	✗
Cueing	✓	✗	✓

Addition – Subtraction



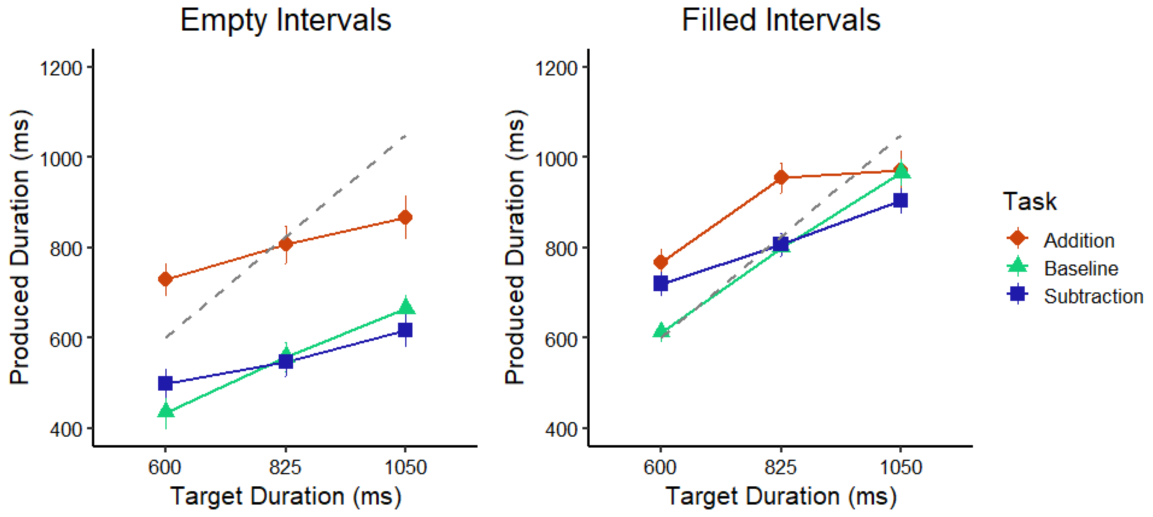
Prado & Knops (*Psych Bull Rev*, 2024)

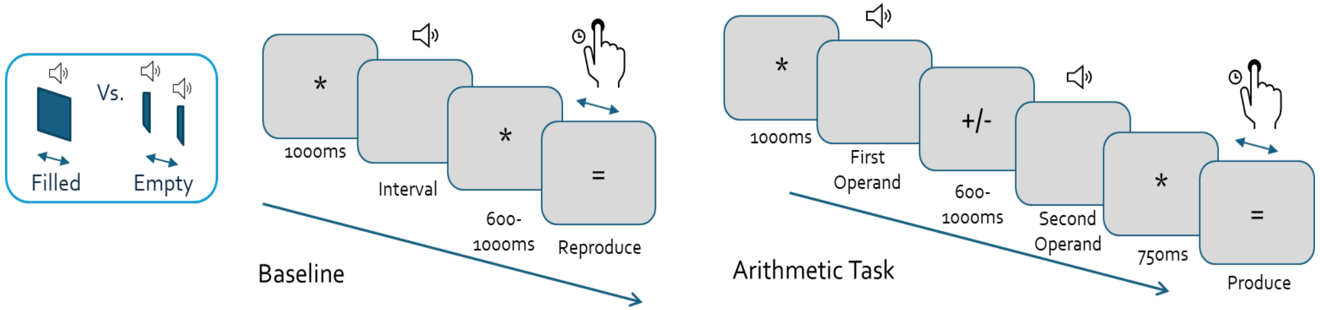


Bonato et al., 2021

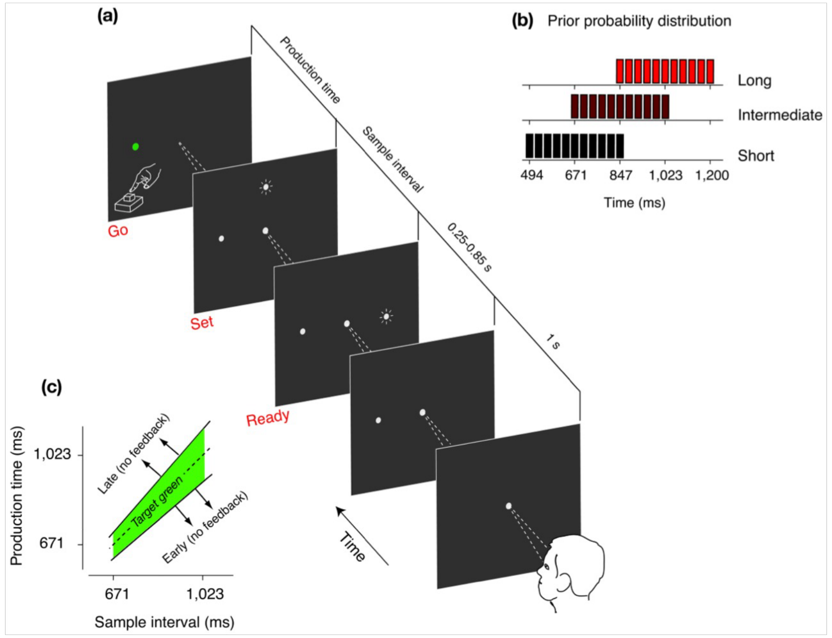
Marie Jacquel

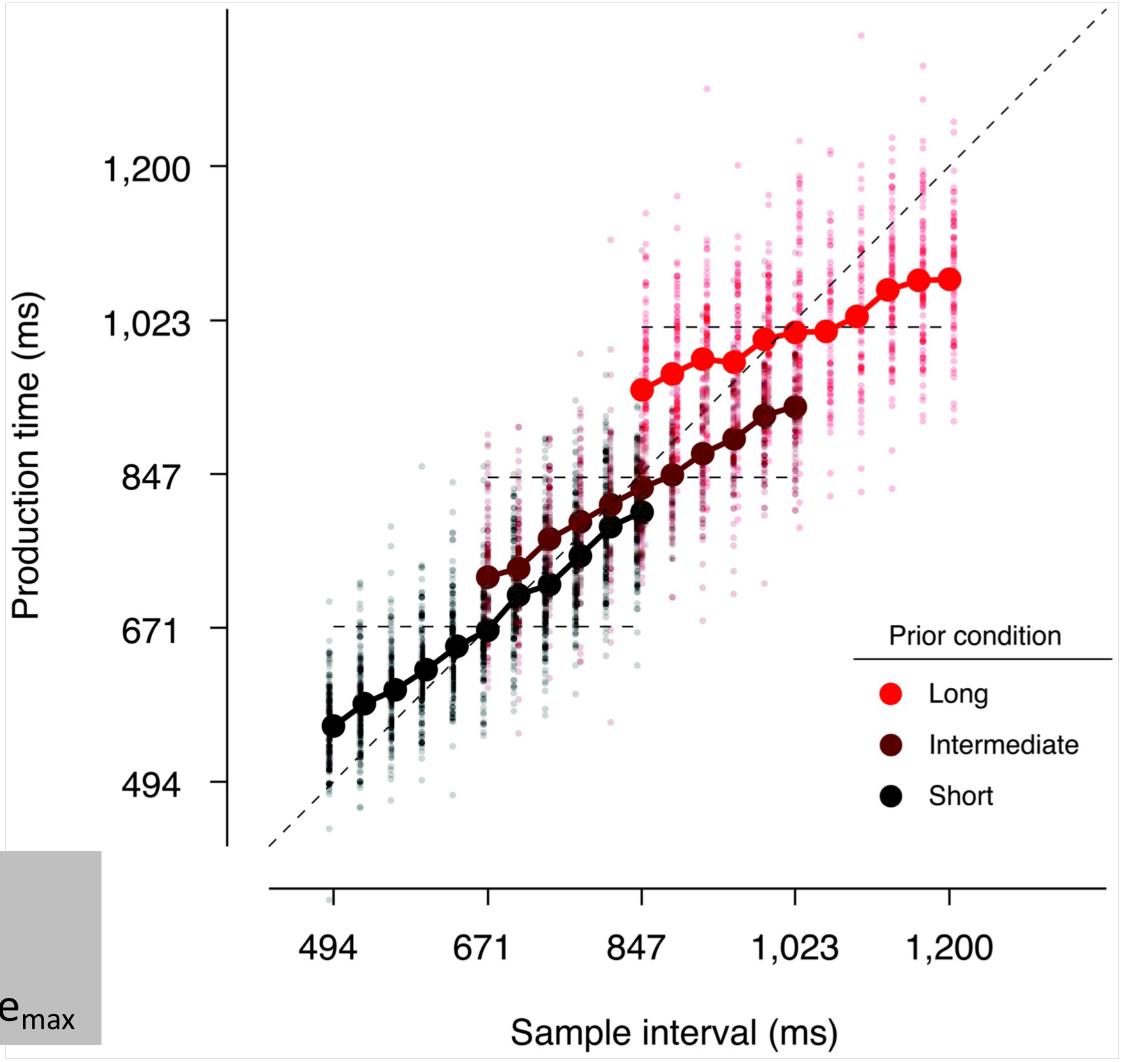
Exp 1





Jacquel, Viarouge & Knops, 2026 *Cognition*





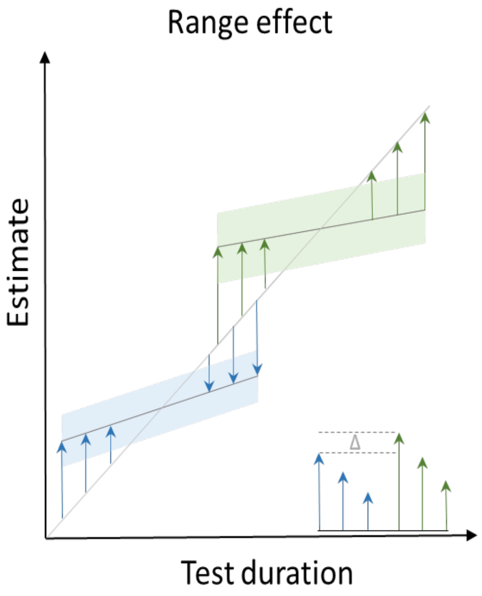
Regression to the mean effect increases

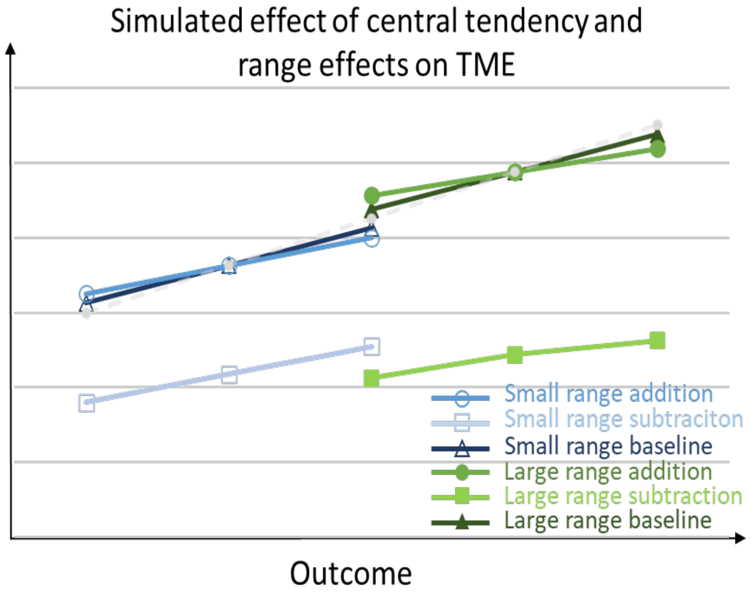
- As a function of the mean overall range
- The larger the distance between $range_{min}$ and $range_{max}$

Jazayeri & Shadlen, Nat Neurosci 2010

Applying the regression to the mean to stimulus ranges in addition and subtraction, we would predict a TME that would increase with mean range

Marie Jacquel





$duration_{est} = a + b * duration_{act}$

The intercept a was computed according to Eq. (A2).

$b = mean(operands\ range) - b * mean(operands\ range)$

Table A1
Parameters for Eq. (A1) that were applied for simulating the TME as shown in Fig. 1.

	Low range	High range
Addition		
Mean(operand range)	362.5 ms	487.5 ms
a	145 ms	231.3 ms
b	0.6	0.5
Subtraction		
Mean(operand range)	737.5 ms	854.2 ms
a	295 ms	427.1 ms
b	0.6	0.5

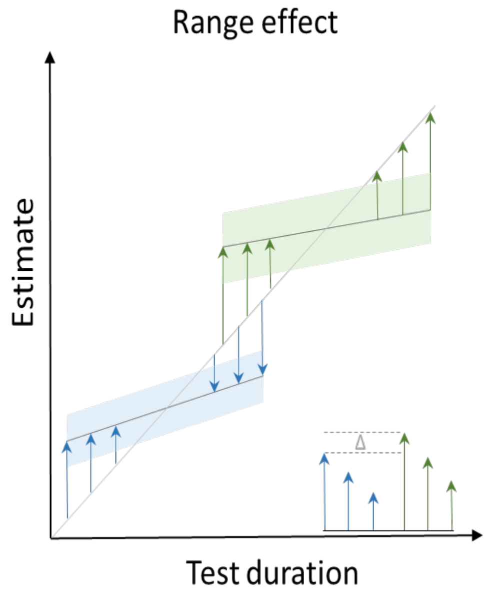
Table A2
Estimated target durations when applying the above equations to four problems from Experiment 2.

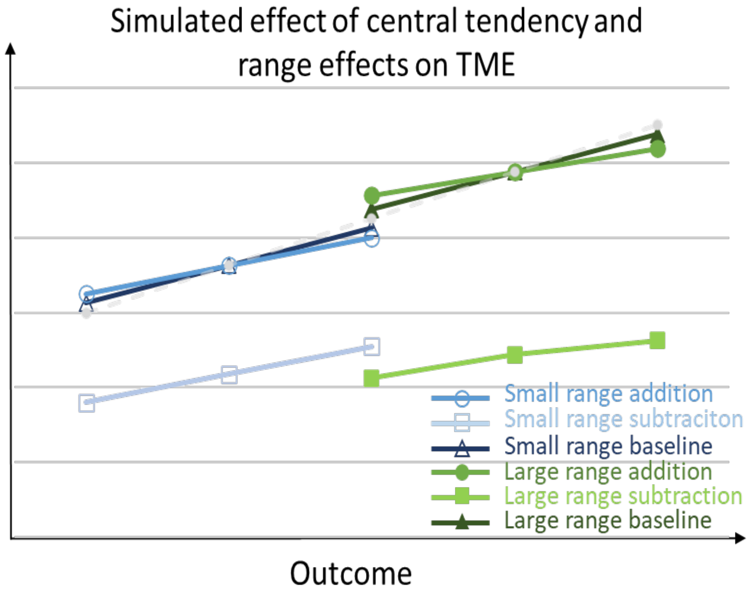
	$Duration_{act\_1}$	$Duration_{act\_2}$	$Duration_{est\_1}$	$Duration_{est\_2}$	Target $Duration_{est}$
Addition					
Low range	600	125	505	220	725
High range	825	150	656	319	975
Subtraction					
Low range	1025	300	910	475	435
High range	1275	300	1065	577	488

Jacquel, Viarouge & Knops, 2026 Cognition

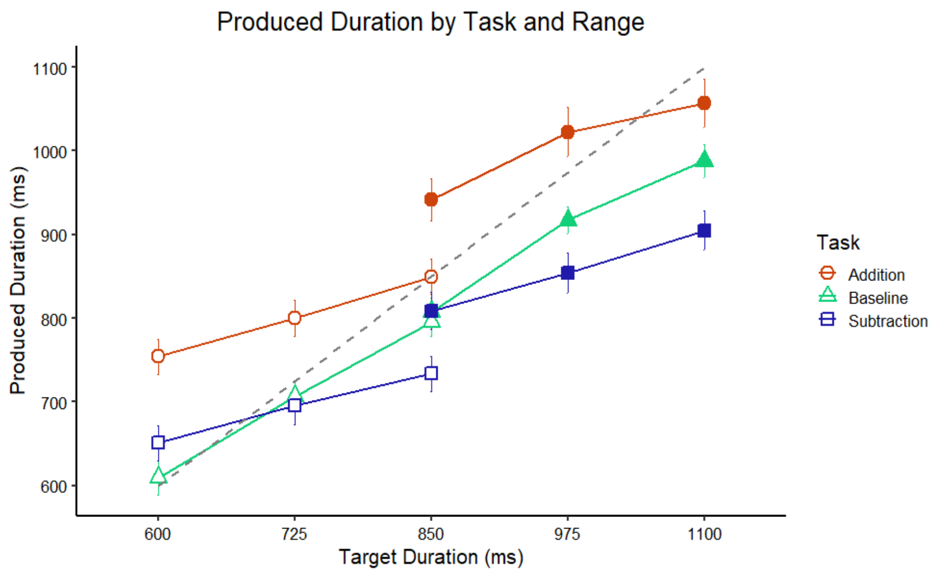
Jacquel et al., *Cognition* 2026

We observe a TME but the results do not follow the predicted pattern.
Regression to the mean not the only source of the TME.





Exp 2



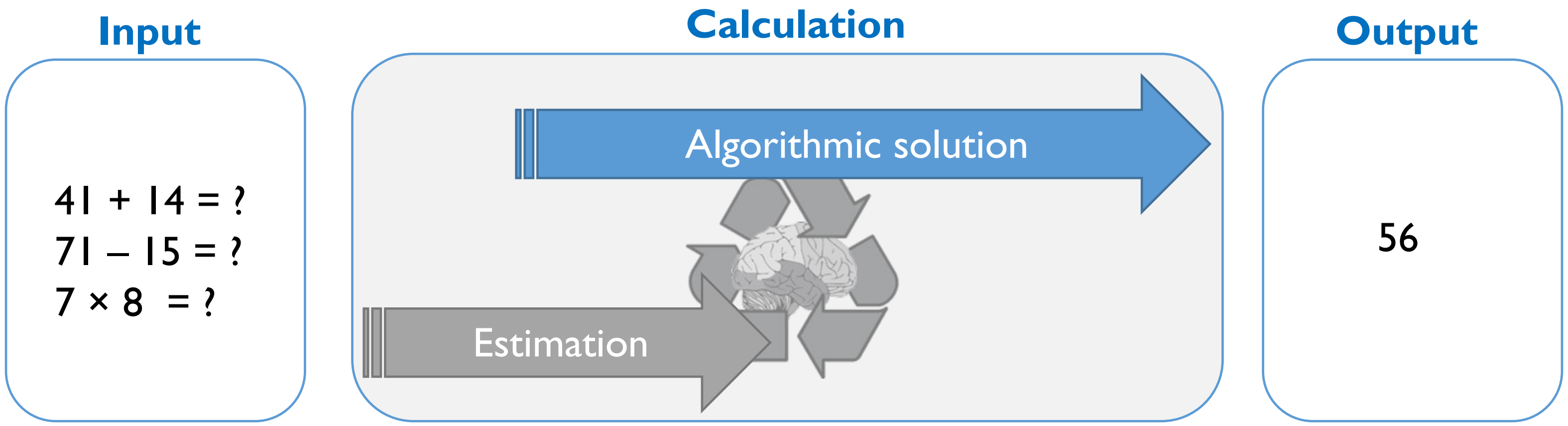
Jacquel, Viarouge & Knops, 2026 Cognition

Jacquel et al., *Cognition* 2026

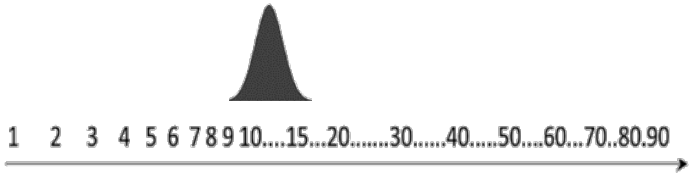
Numerical Cognition

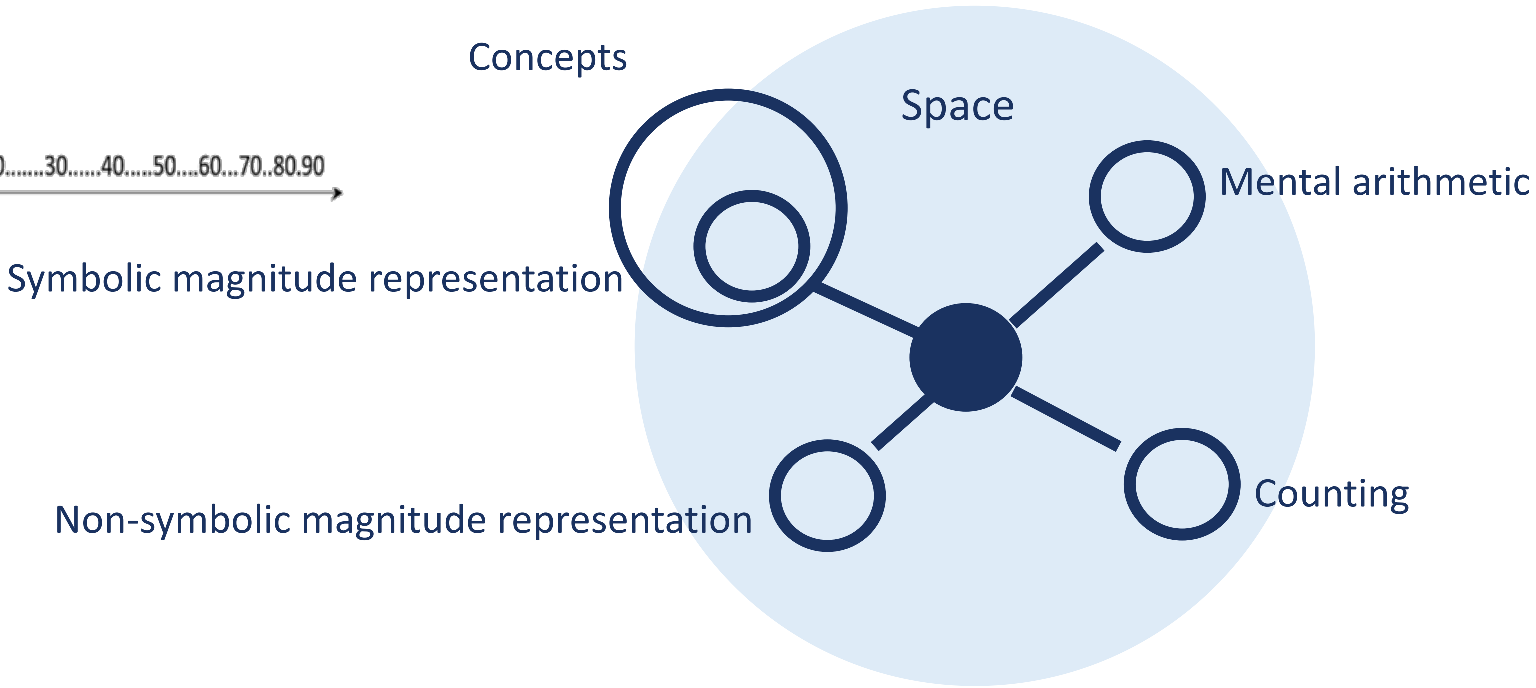
Processes Underlying **Mental Arithmetic**

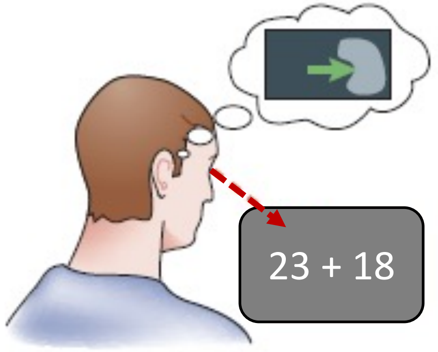
- Mental arithmetic invokes visuo-spatial mechanisms.
  - Left saccade associated to subtraction
  - Right saccade associated to addition
- Temporal arithmetic subject to comparable biases



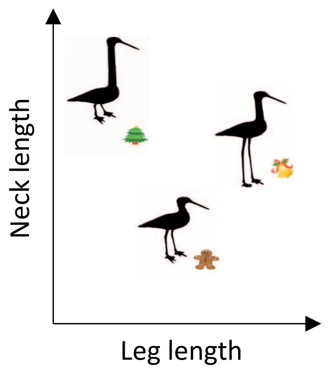
Numerical cognition has many facets



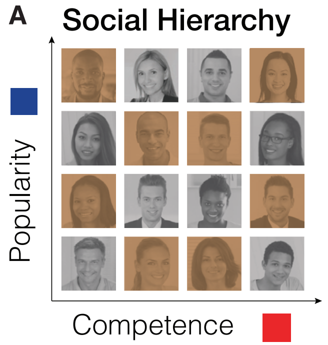




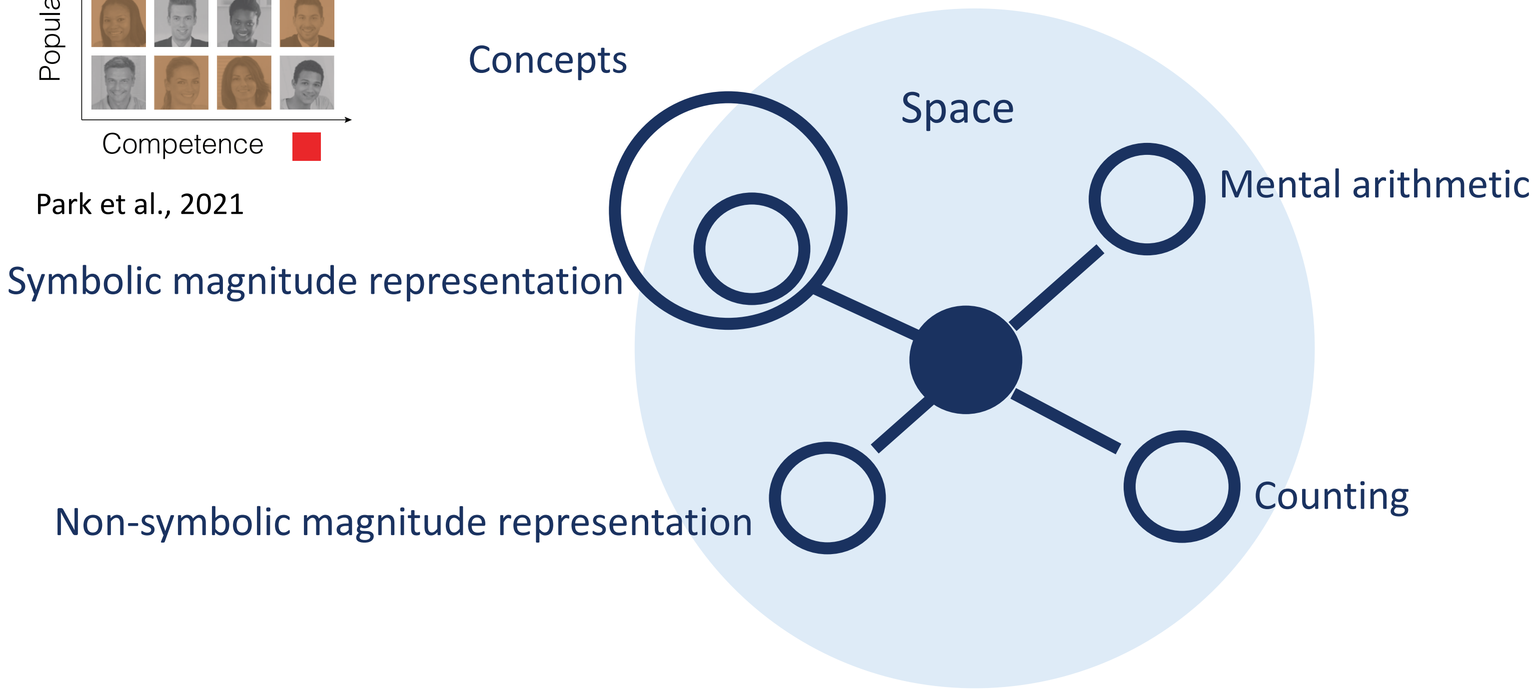
Numerical cognition has many facets

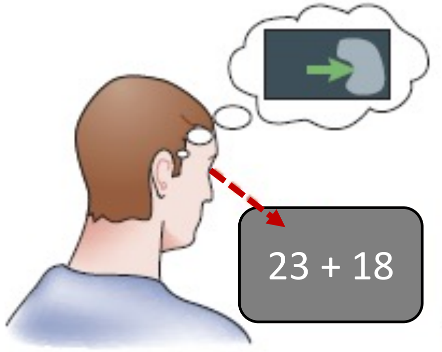


Constantinescu et al., 2016

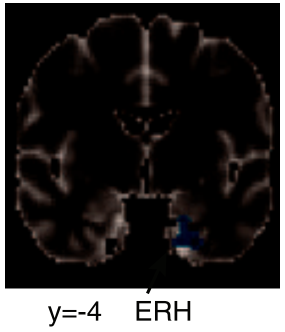


Park et al., 2021

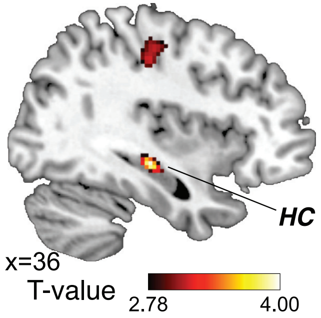




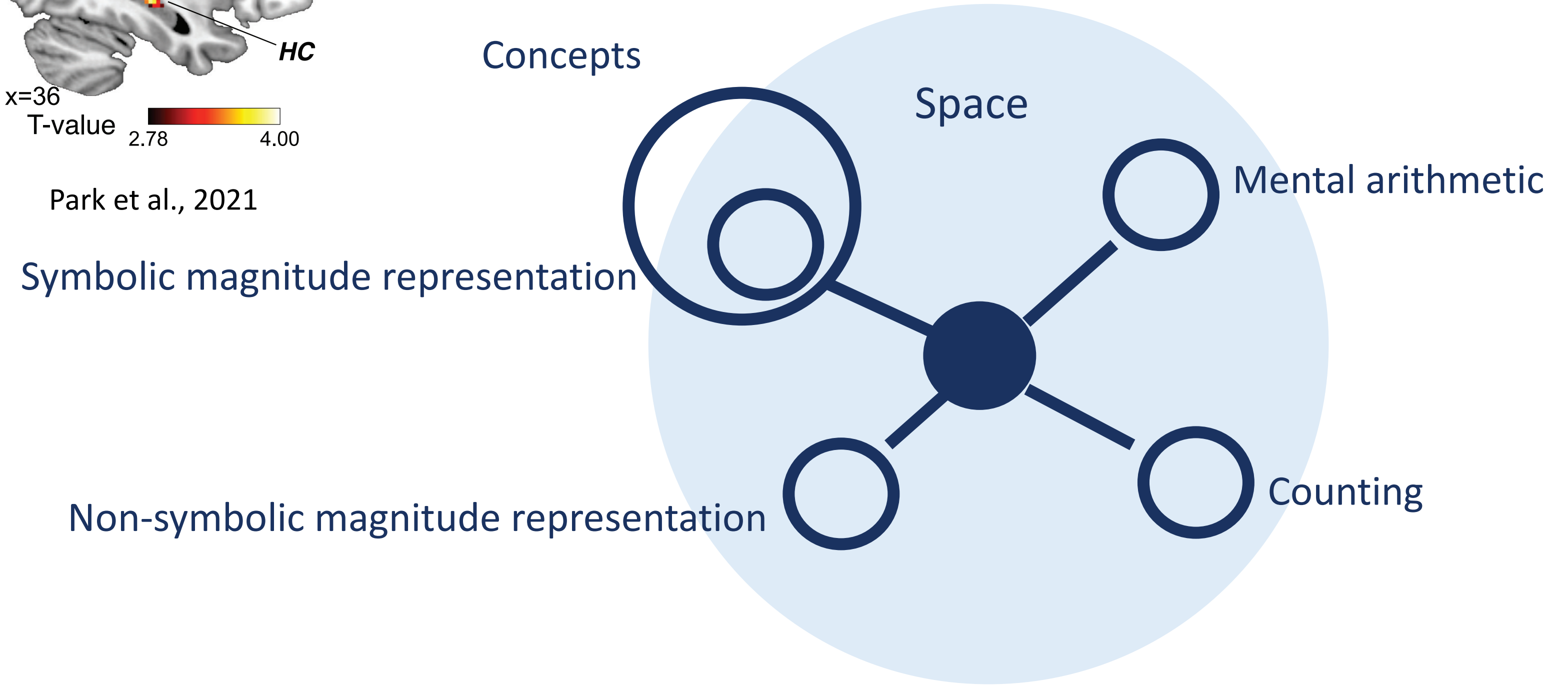
Numerical cognition has many facets

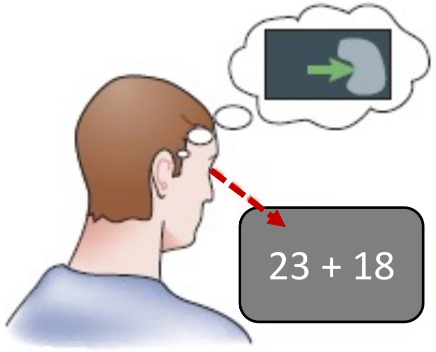


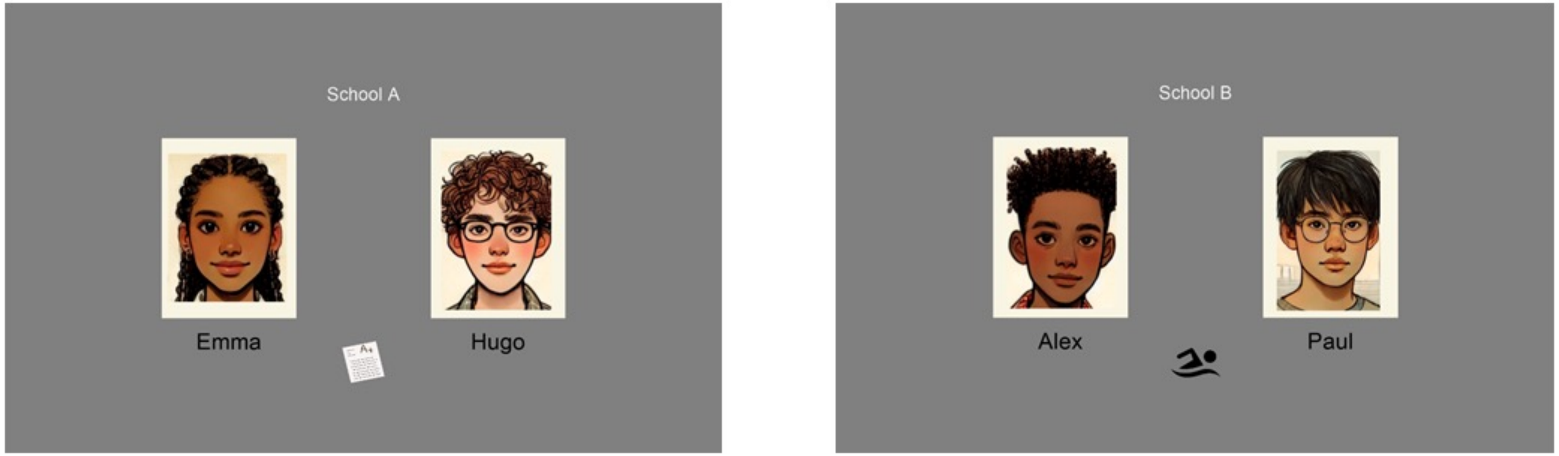
Constantinescu et al., 2016

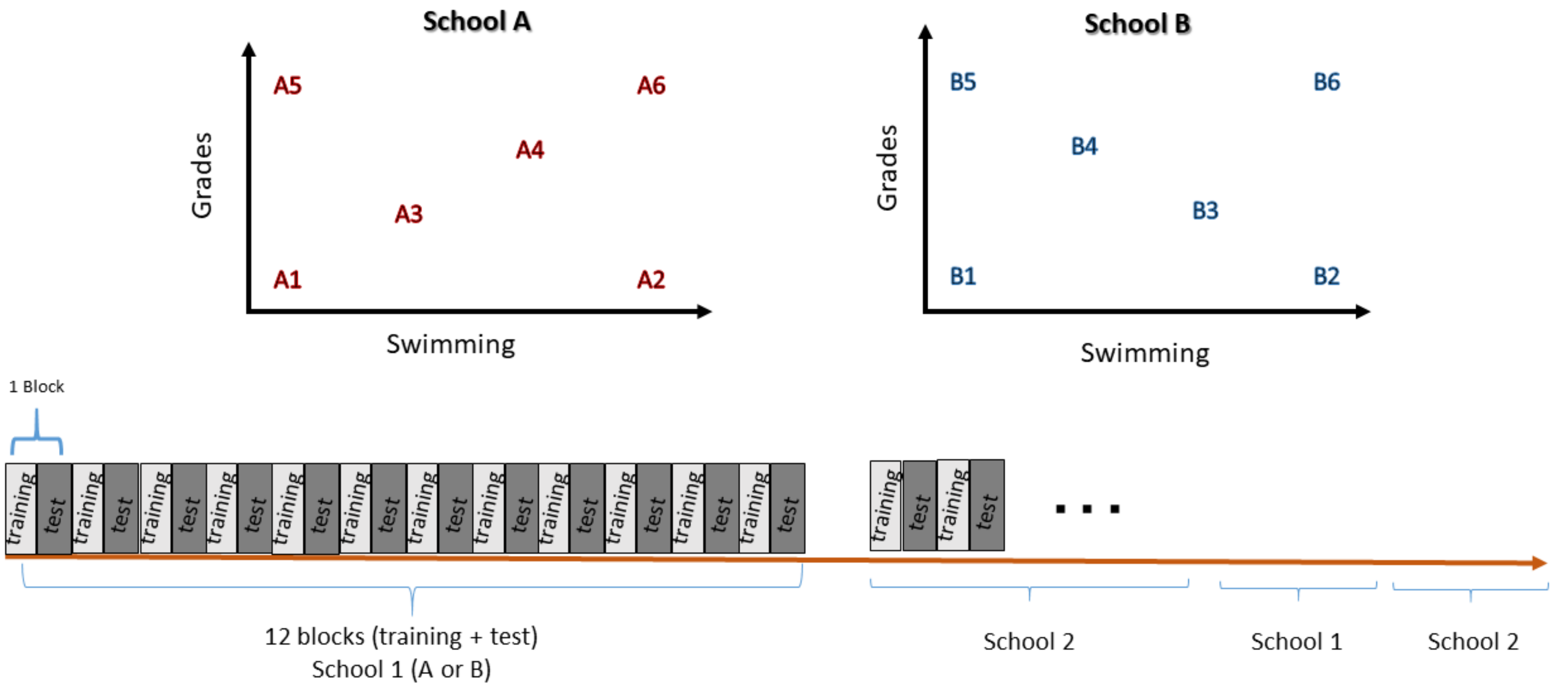


Park et al., 2021



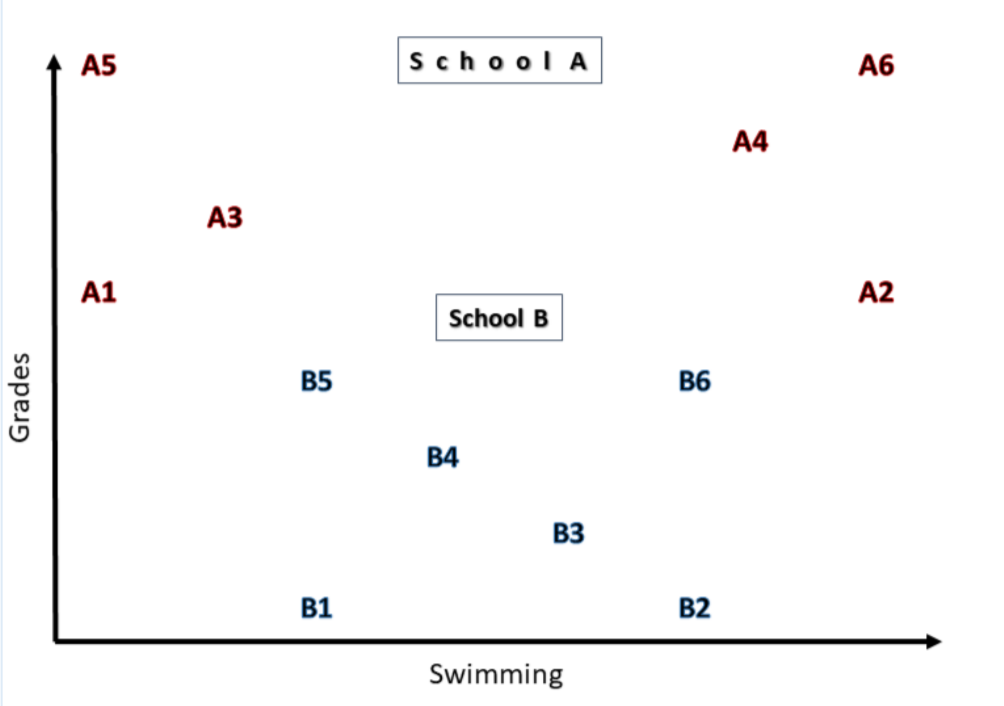


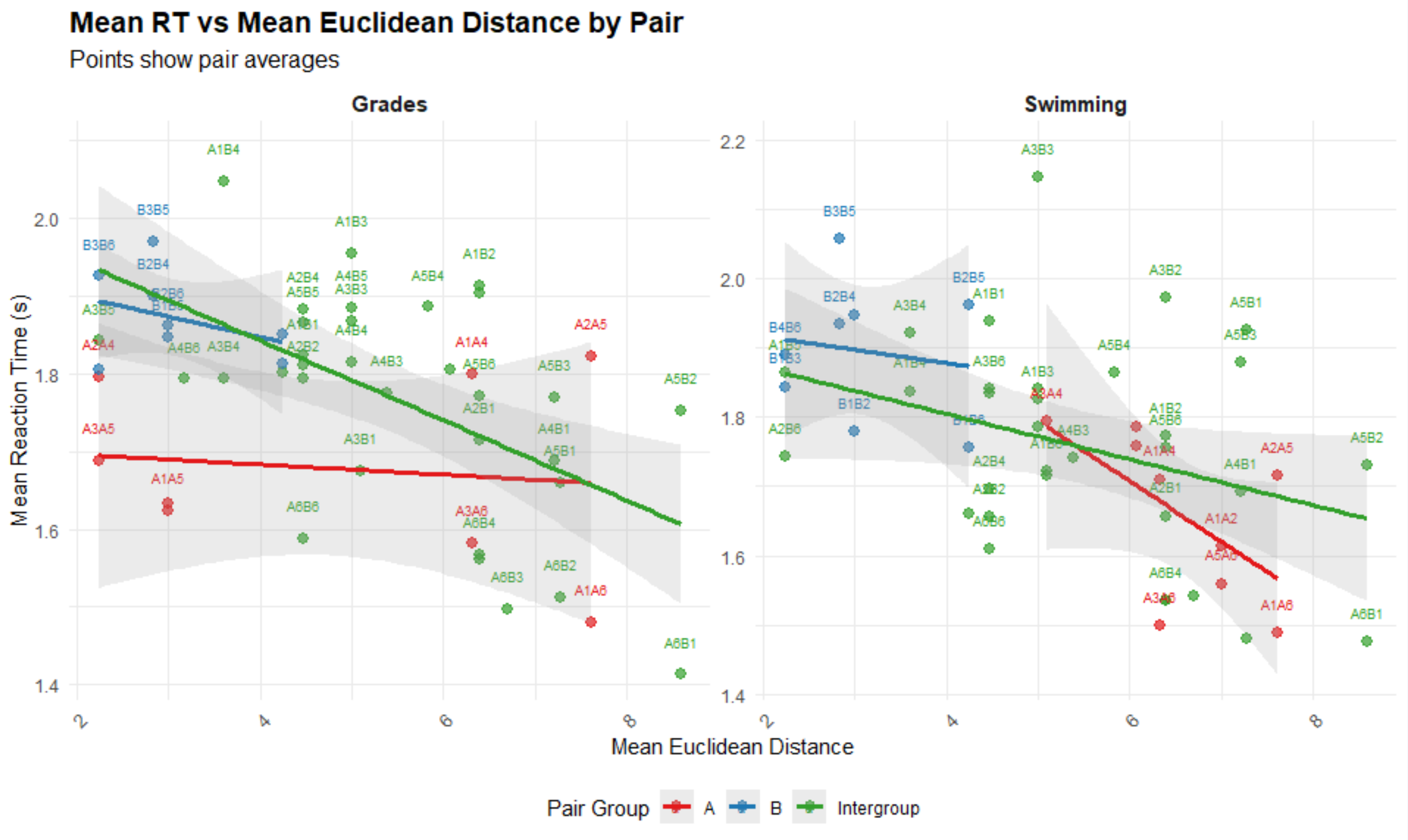




Gözde Vatansever

Vatansever, Fischer & Knops , in preparation



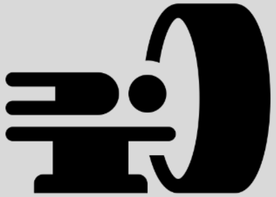


Group A:

- intercalation (swimming) lead to an increased distance effect
- Chaining (grades) lead to loss of spatial coding (distance effect)

Group B:

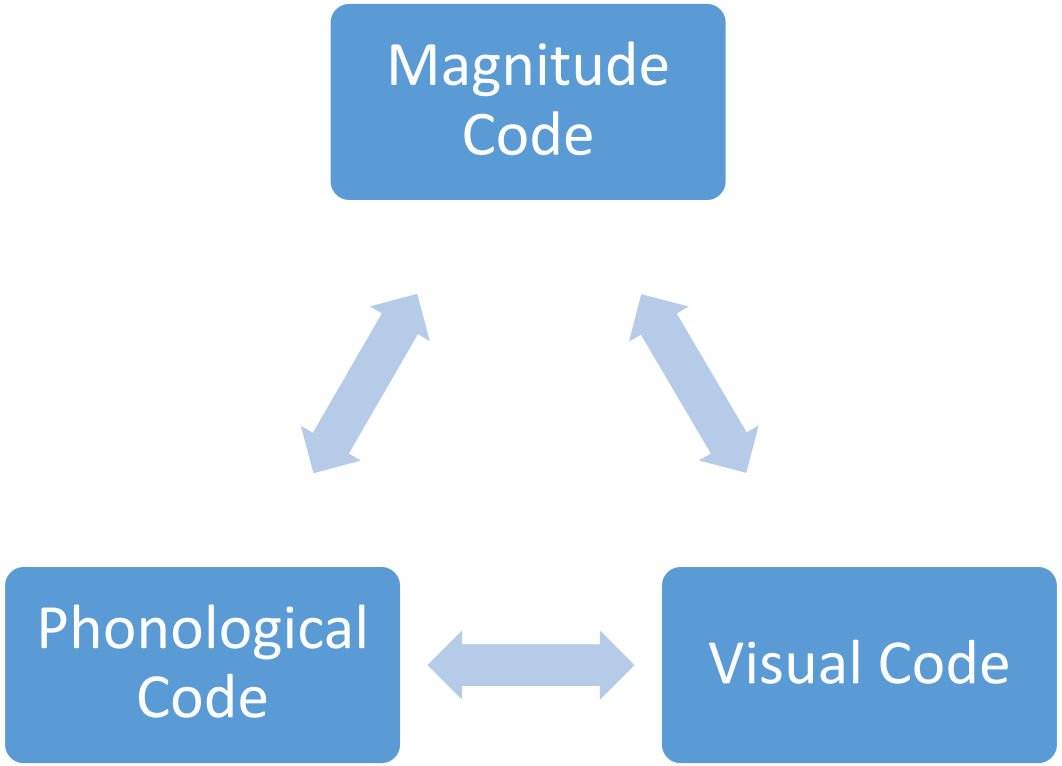
- Loss of spatial coding after merging (distance effect)



ongoing

Vatansever, Fischer & Knops , in preparation

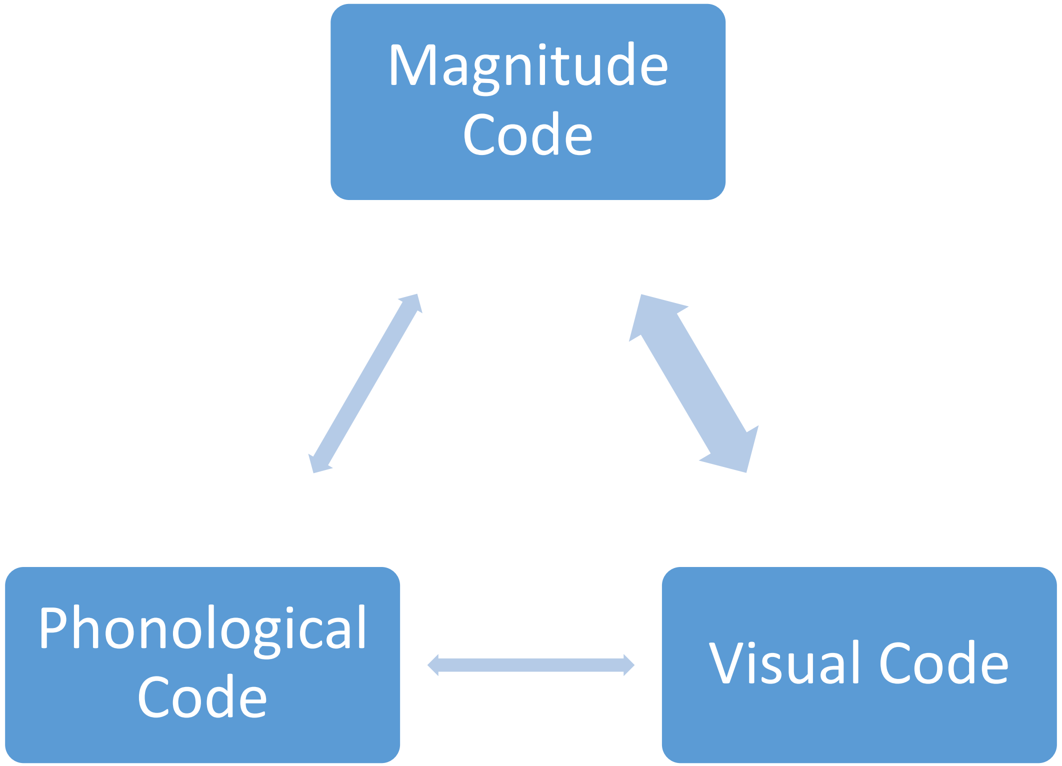
- Integration reduced the discriminability of distances
- Spatial coding depends on the geometry of reorganisation rather than the domain or individual capacity.
- Cognitive maps as **adaptive** representational structures shaped by current task demands and structural context



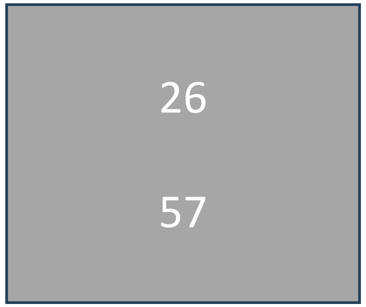
Dehaene & Cohen, 1995

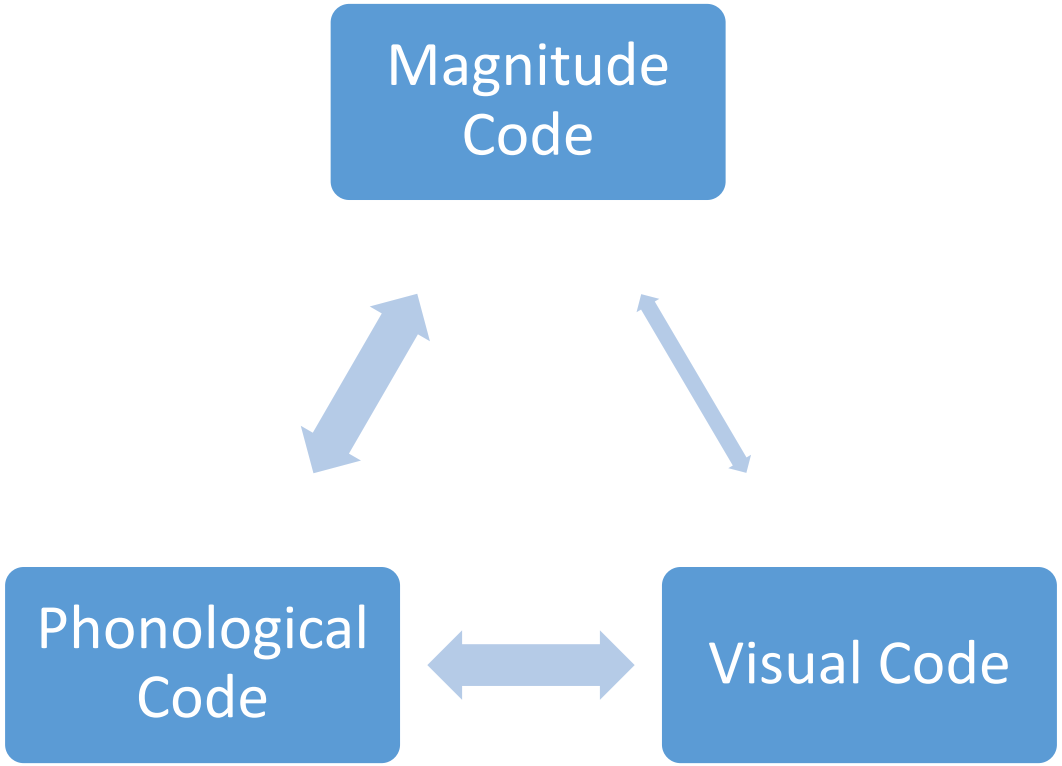
26

57

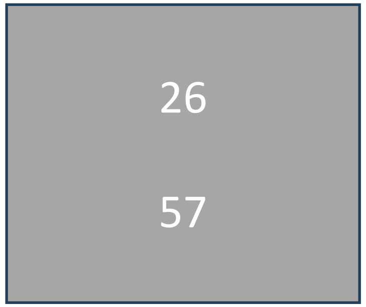


Dehaene & Cohen, 1995



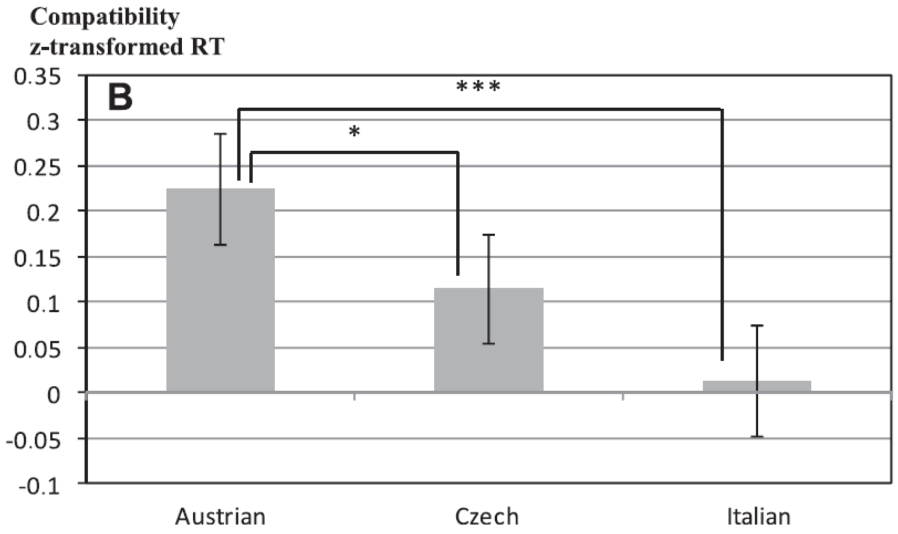


Dehaene & Cohen, 1995



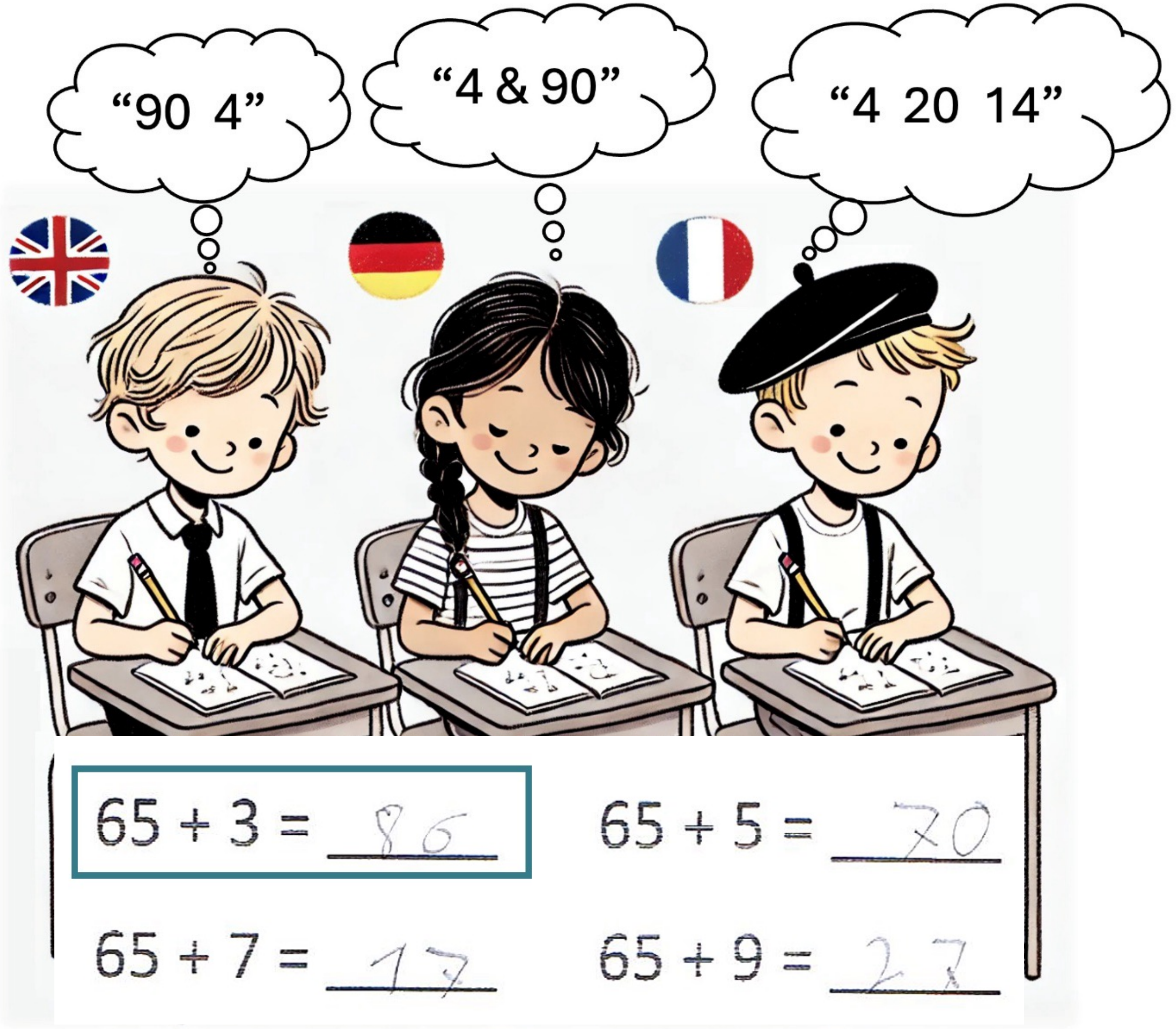
Interrelation of numerical cognition and language

- ✓ Magnitude comparison → no verbal processing required
- ✓ Unit-decade compatibility effect (UDCE) → interference from irrelevant unit digit
- ✓ Modulation of the UDCE by language
  → Arabic digits transparent, universal (value determined by place of digit), verbal number word systems vary with language



Pixner et al. (2011)

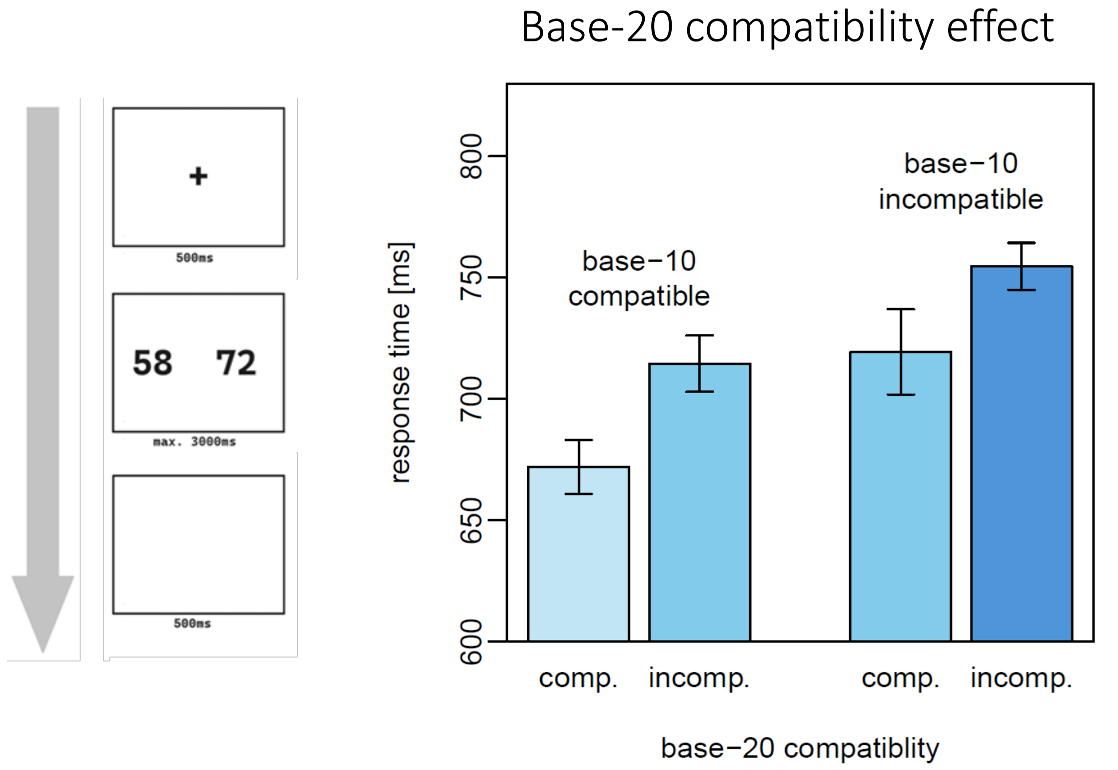
Interrelation of numerical cognition and language

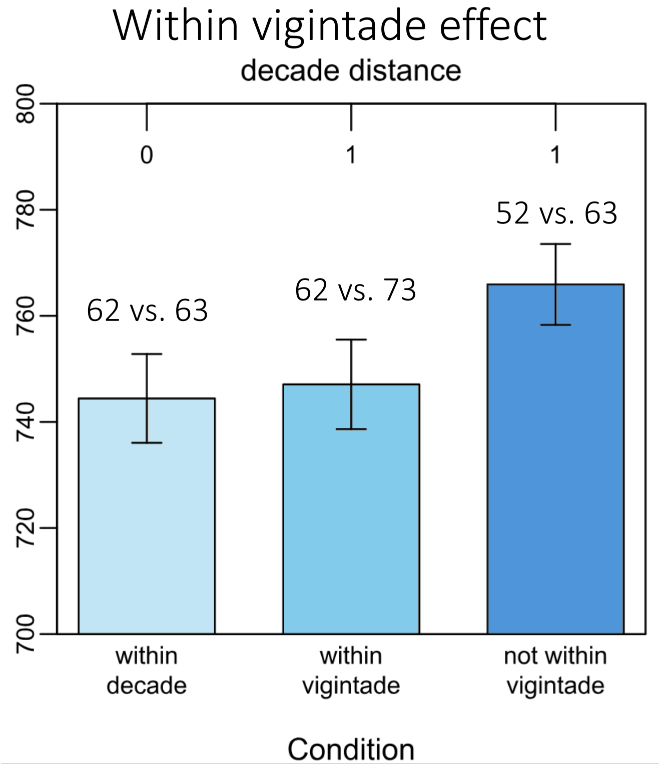


→ Utilization of the discrepancy between Arabic numeral notation (95 = 90_5) and French number-word syntax (85 = 4_20_15)
→ Linguistic effects that can clearly be attributed to vigesimal number words?

Interrelation of numerical cognition and language

Elise Klein Roman Janssen





- ✓ Magnitude comparison > 60 in French speakers: Base-10 and base-20 effect in parallel
- ✓ Additional within-vigintade effect not affected by confound between base-20 compatibility and decade distance

„… verbal representations can shape basic numerical judgments and that number processing may be more closely tied to language than previously assumed.“

Janssen et al., Open Mind (2026)

Task-specific factors

- stimulus set
- 1D vs. 2D
- Dominance vs. Magnitude
- …

Language

- Inversion
- Number word syntax
- …

Concepts