How do we learn that 2 + 3 = 5?

Behavioral and neural evidence for procedural automatization of arithmetic during development

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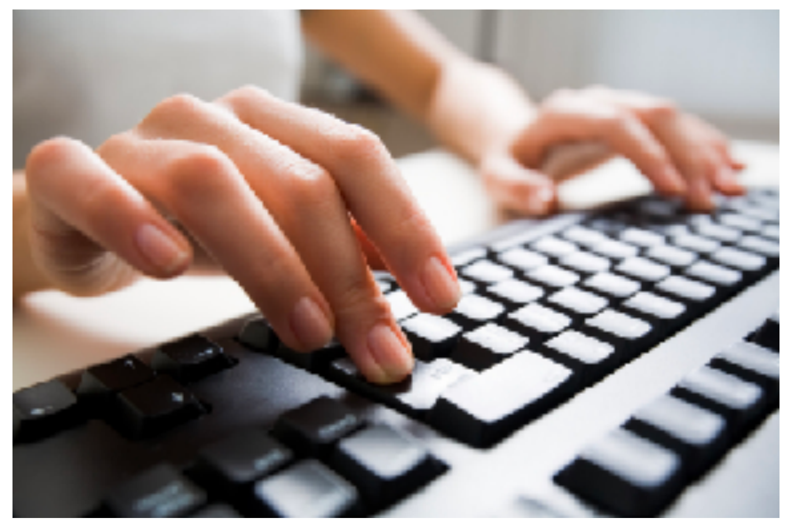
France

INT, Marseille – July 6 2023

Automaticity in the mind

- Doing simple arithmetic is fast, effortless, autonomous, and unconscious

→ Automatic



Shiffrin & Schneider (1977), Psych. Review

Logan (1988), Psych. Review

How does a task become automatic?

"automaticity is memory-based processing and automatization is a shift from algorithmic processing [...] to memory retrieval"

Logan (1997)

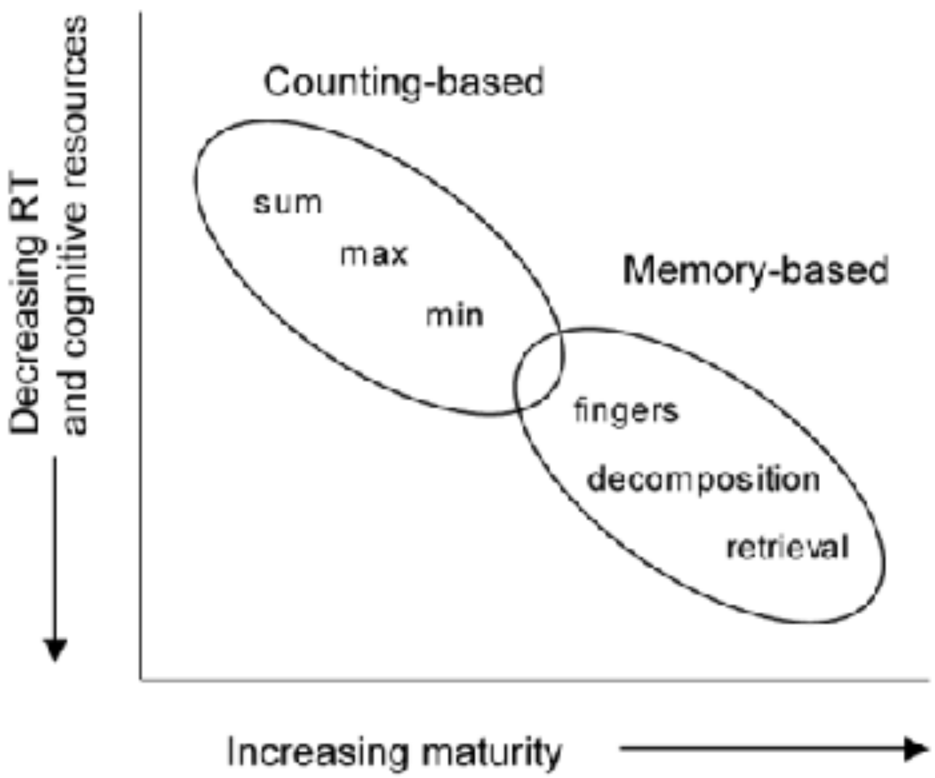
- A novel task initially requires effortful mental computation
- Each instance of encountering the task creates a memory trace or strengthens the connection between stimulus and response
- Automatic tasks rely on retrieving associations from memory

Objects or thoughts that are experienced concomitantly become progressively associated in memory

Shiffrin & Schneider (1977), Psych. Review

Logan (1988), Psych. Review

Mental arithmetic as a case study of automatization



- Solving simple arithmetic problems repeatedly should lead to an association between operands and answer
- This association may be verbal in nature
- Solving simple arithmetic problems should not involve access to number magnitude in expert individuals

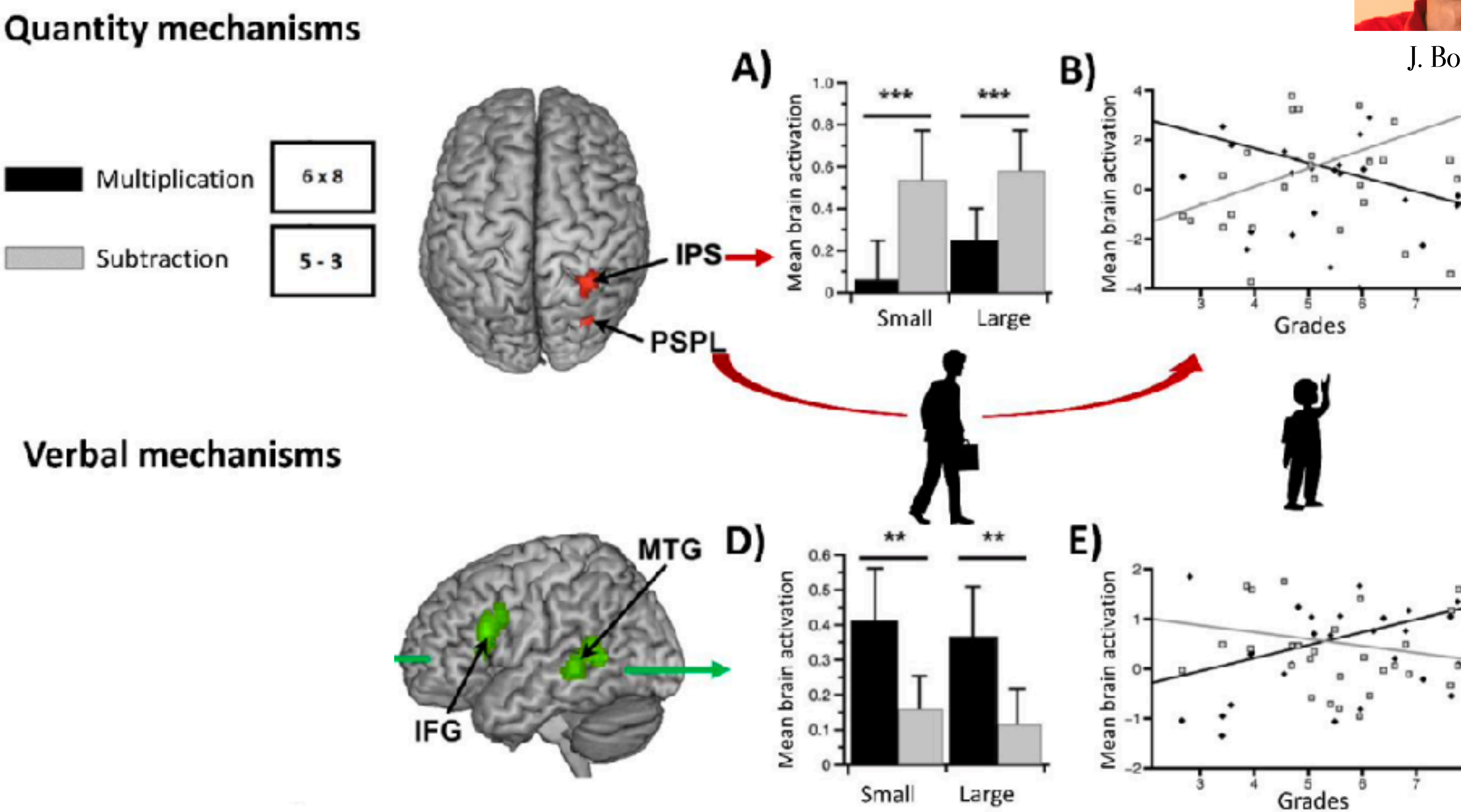
Dehaene and Cohen (1995) Mathematical cognition

Geary & Hoard (2005) Handbook of mathematical cognition

Are arithmetic facts retrieved from verbal memory?

J. Booth

n = 26 adults   n = 34 children



Prado, Mutreja, & Booth (2014) Dev. Science

Prado, Mutreja, Zhang, Mehta, Desroches, Minas, & Booth (2011) HBM

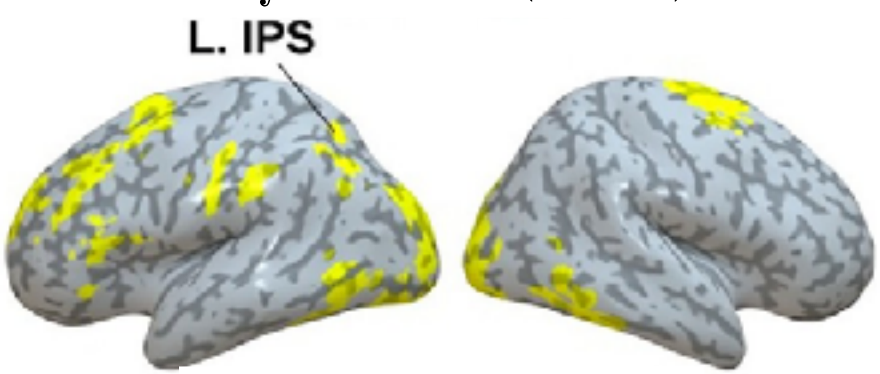
Are arithmetic facts retrieved from verbal memory?


P. Bhatia

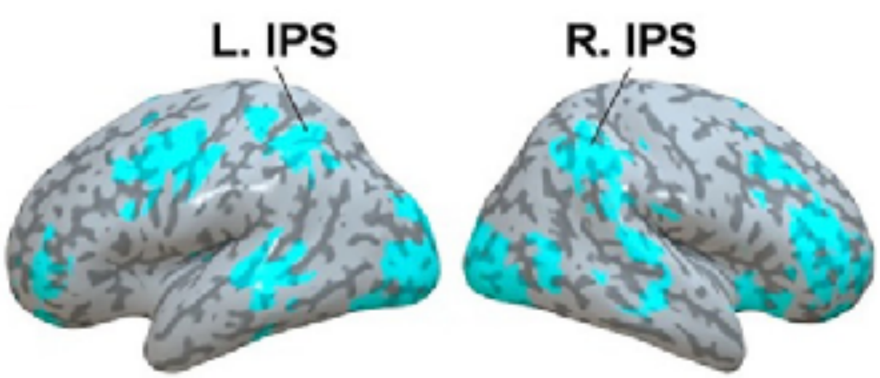

C. Girard

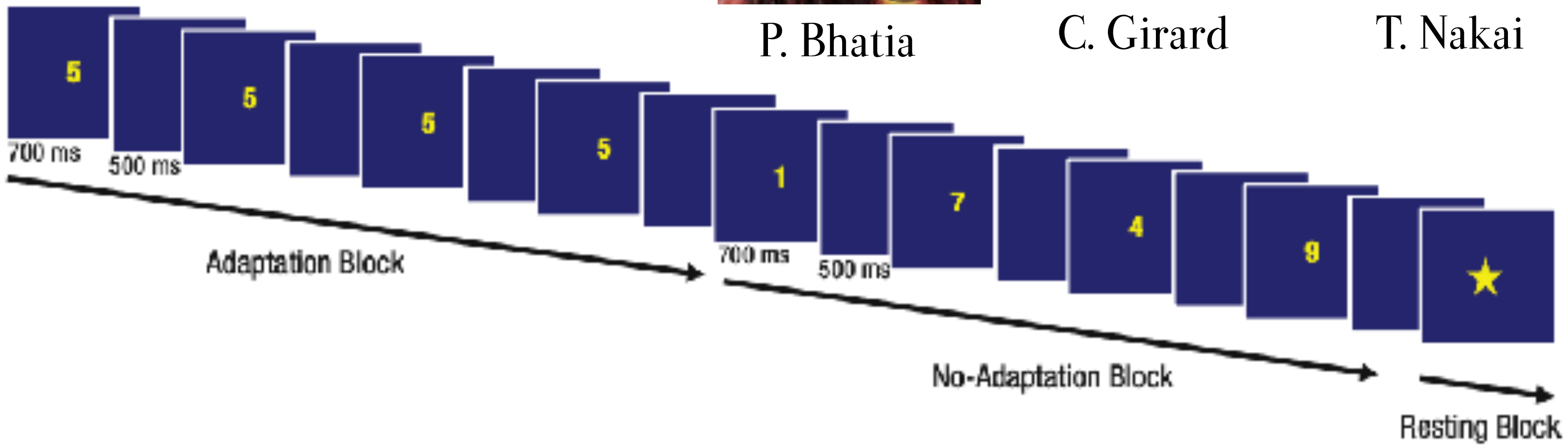

T. Nakai

5-year-olds (n=43)



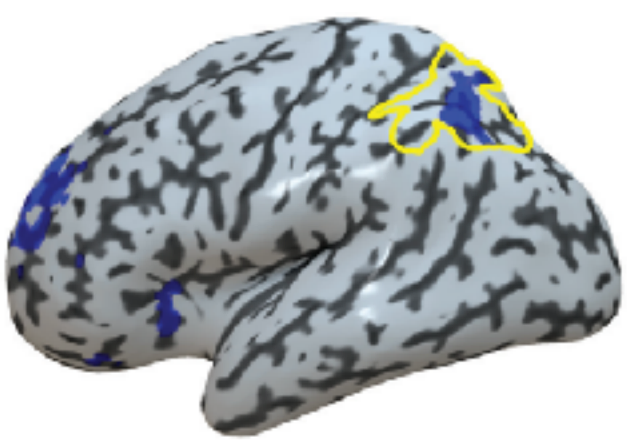
8-year-olds (n=46)

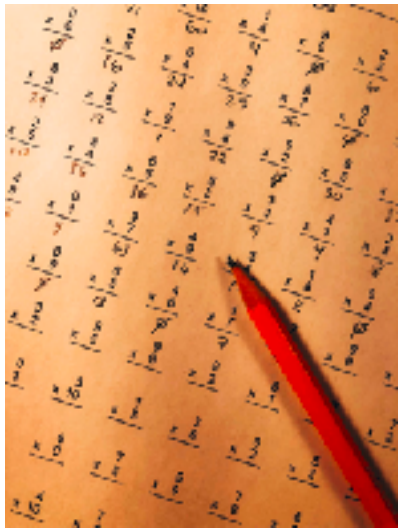




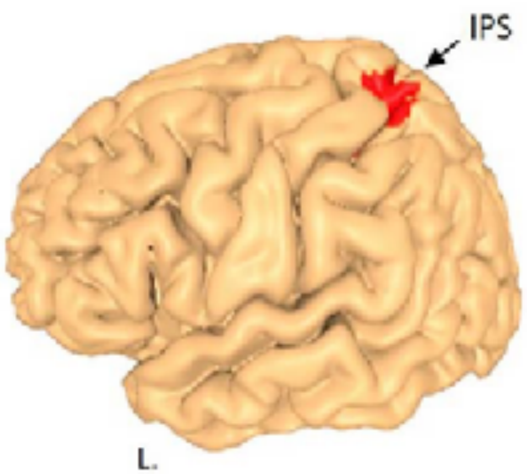
Correlation with arithmetic skills

8-year-olds (n=50)





Adults (n=48)



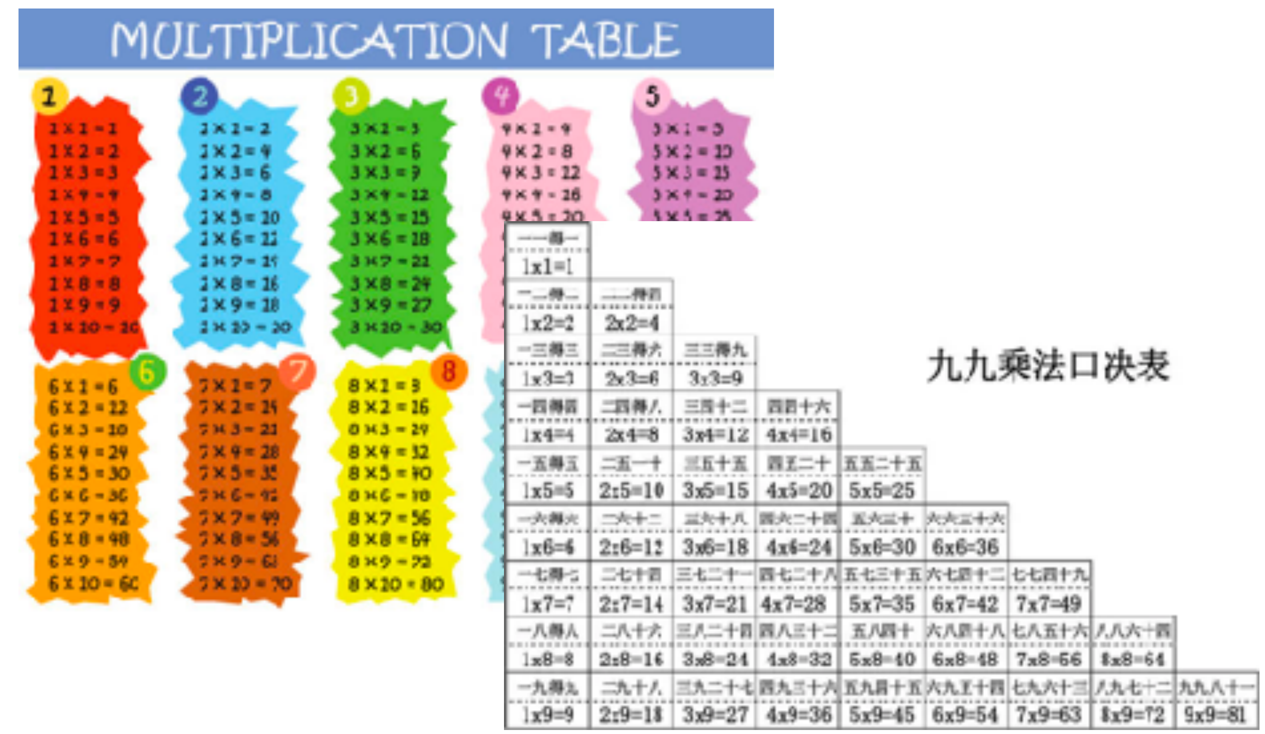
Bhatia, Longo, Chesonokova, & Prado (2022) Cereb. Cortex

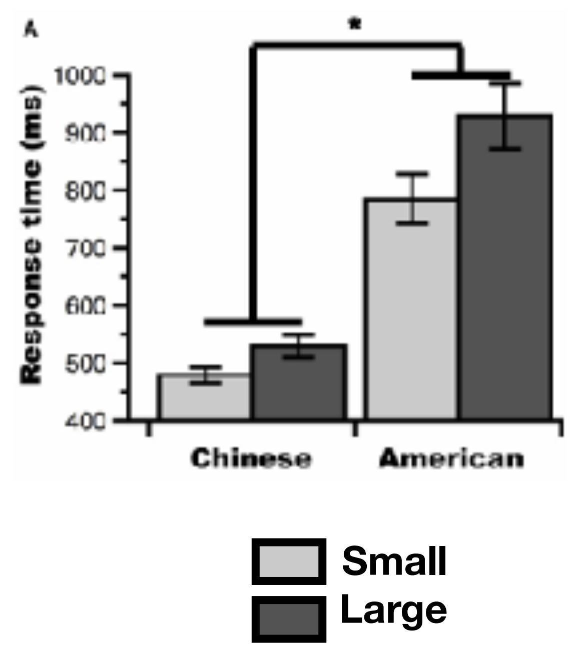
Girard, Bastelica, Léone, Epinat-Duclos, Longo, & Prado (2022) Psych. Science

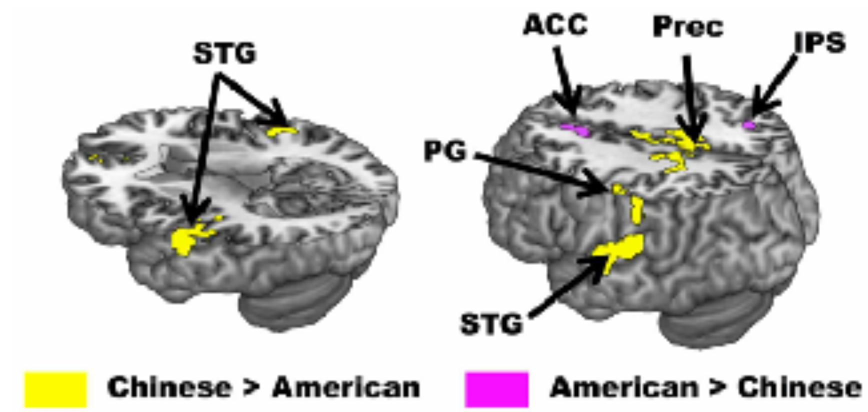
Nakai, Girard, Longo, Chesnokova, & Prado (2023) PLOS Biol.

Are arithmetic facts retrieved from verbal memory?

- Simple arithmetic facts engage parietal mechanisms supporting numerical magnitude
- Single-digit multiplication may be an exception, as it is associated with language mechanisms
- This is likely to be due to teaching strategies emphasizing the explicit learning of multiplication facts







n = 26 adults in US
n = 27 adults in China

Prado, Lu, Liu, Dong, Zhou, & Booth (2013) Front. Hum. Neurosci.

Dissecting arithmetic facts

$$2 + 3 = 5$$

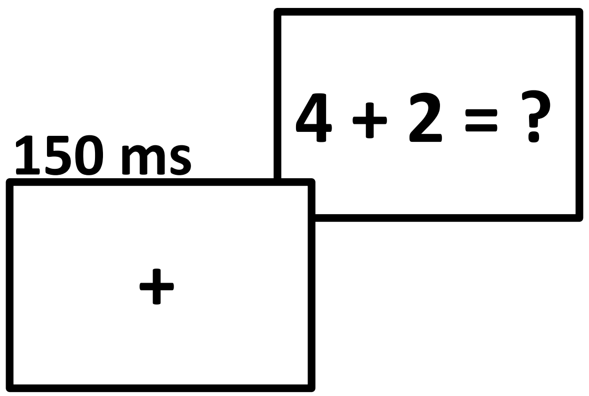
$$2 \textcolor{red}{+} 3 = 5$$

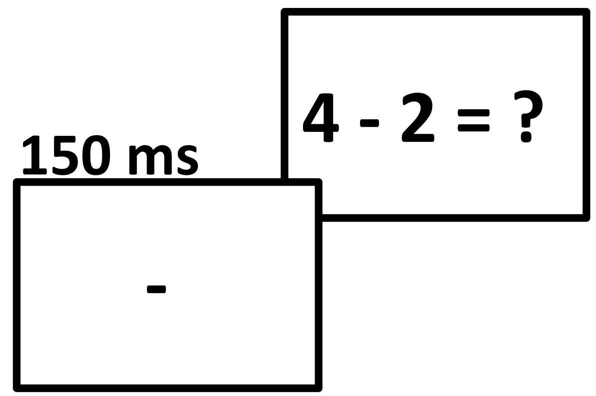
$$2 + 3 = 5$$

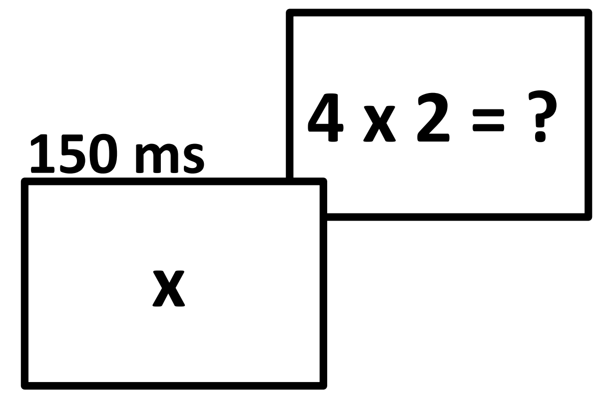
Operator processing

C. Poletti

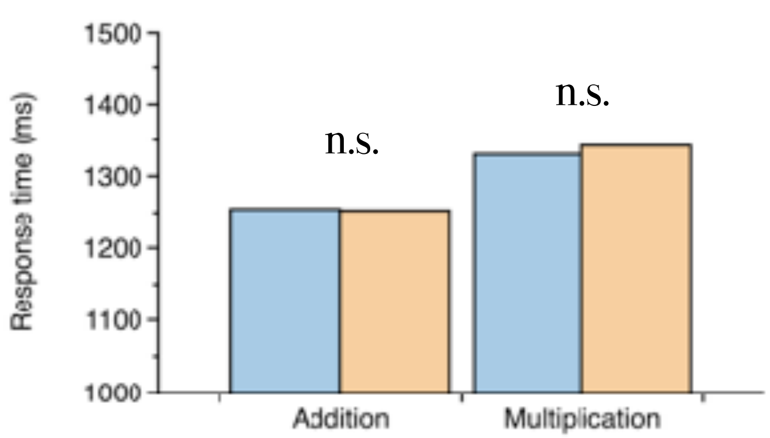
C. Thevenot



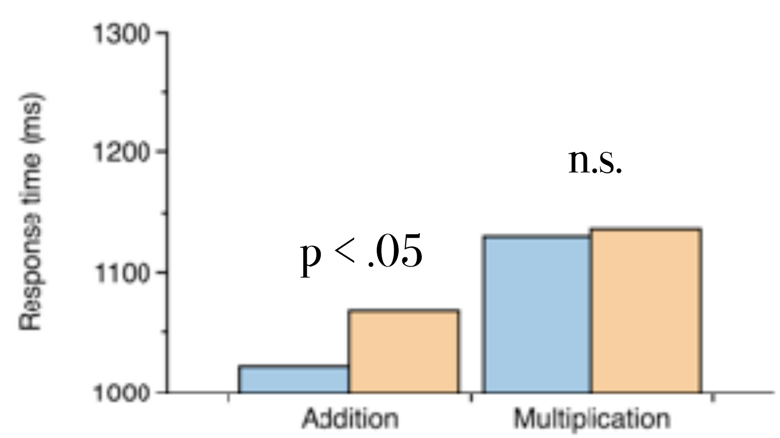




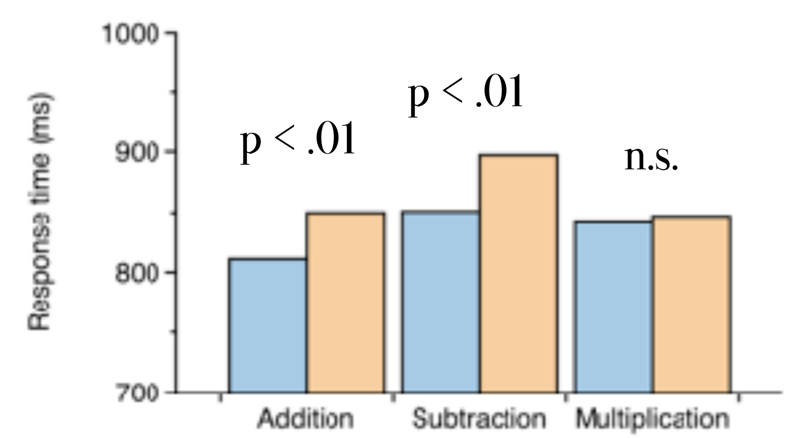
<u>10-12-yo (n = 33)</u>



<u>13-15-yo (n = 28)</u>



<u>Adults (n = 18)</u>



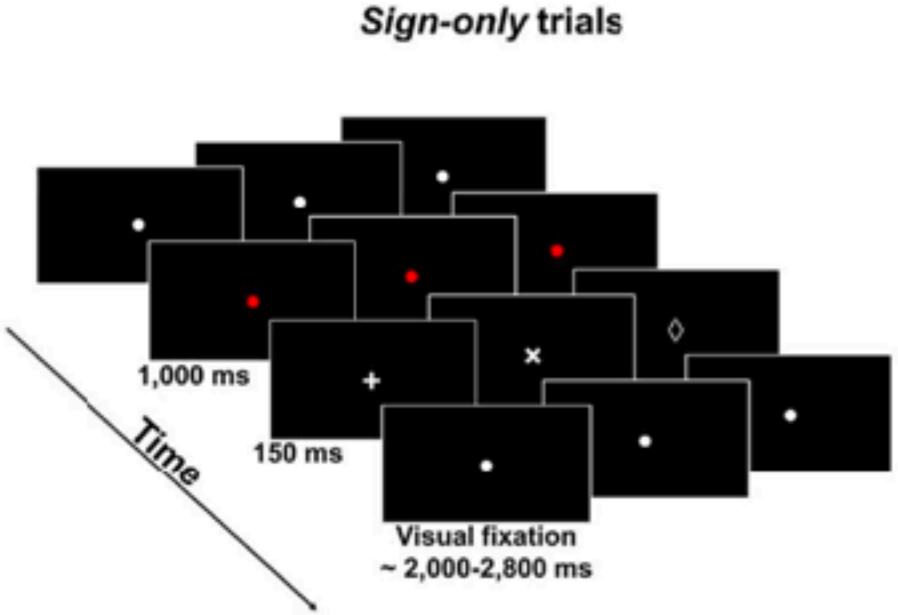
priming
no priming

Fayol & Thevenot (2012) Cognition

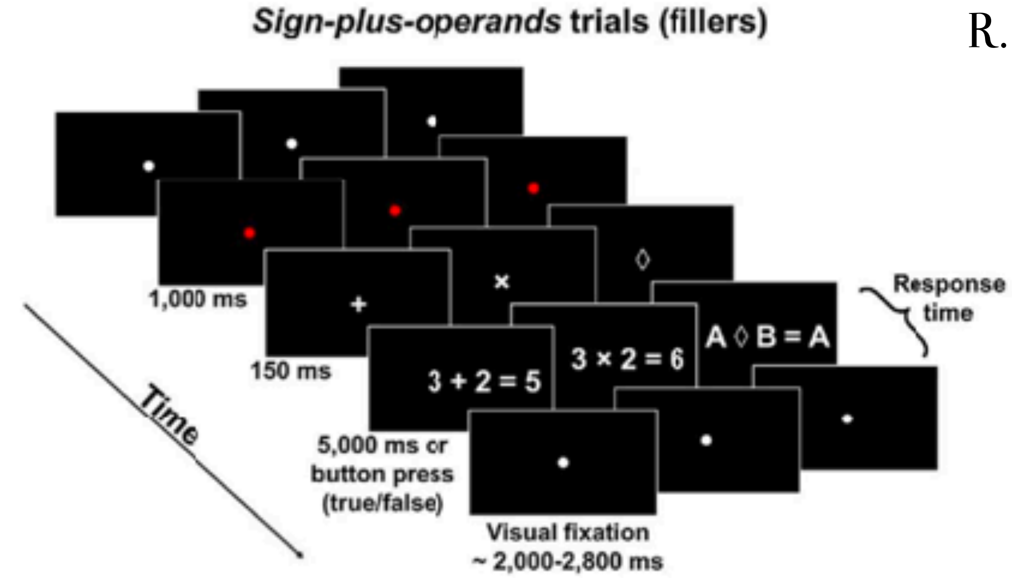
Poletti, Perez, Houillon, Prado, & Thevenot (2021) Br. J. Dev. Psychol.

Operator processing

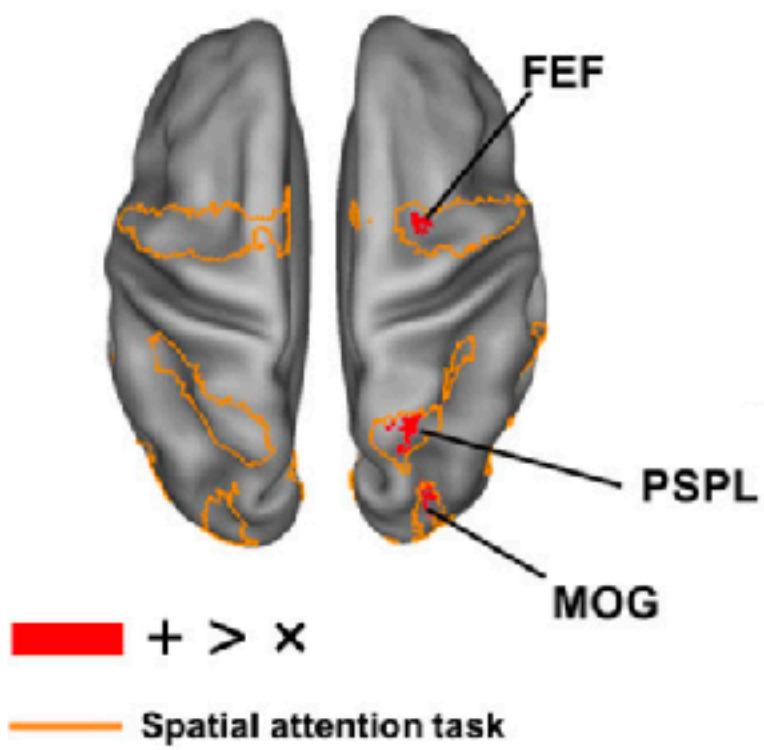
R. Mathieu

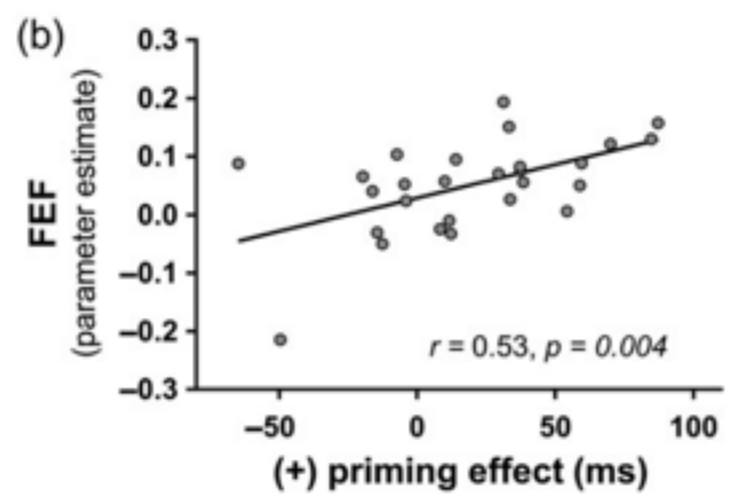


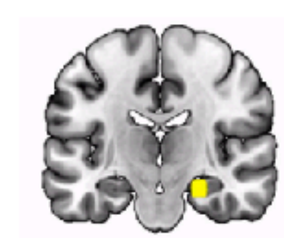
n = 27 adults

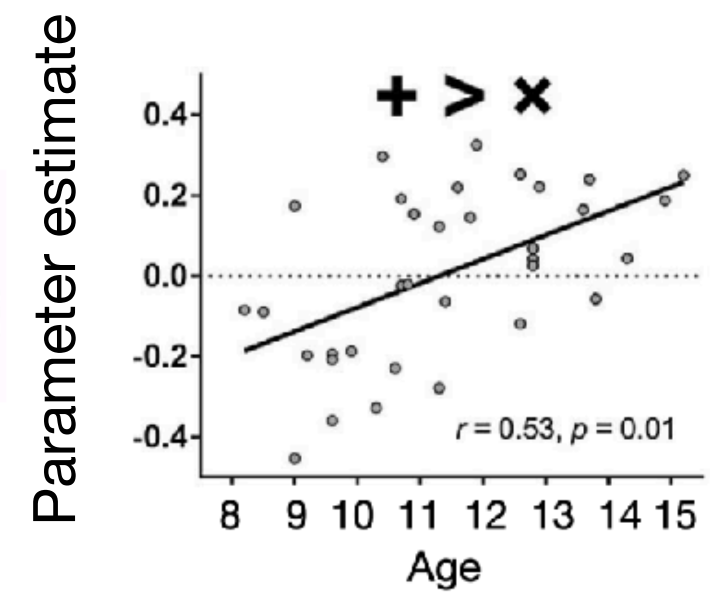


n = 34 children









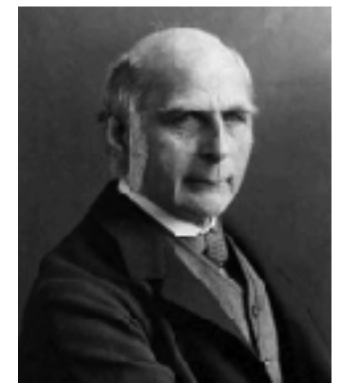
Mathieu, Epinat-Duclos, Léone, Fayol, Thevenot, & Prado (2018) Dev. Cog. Neuro.

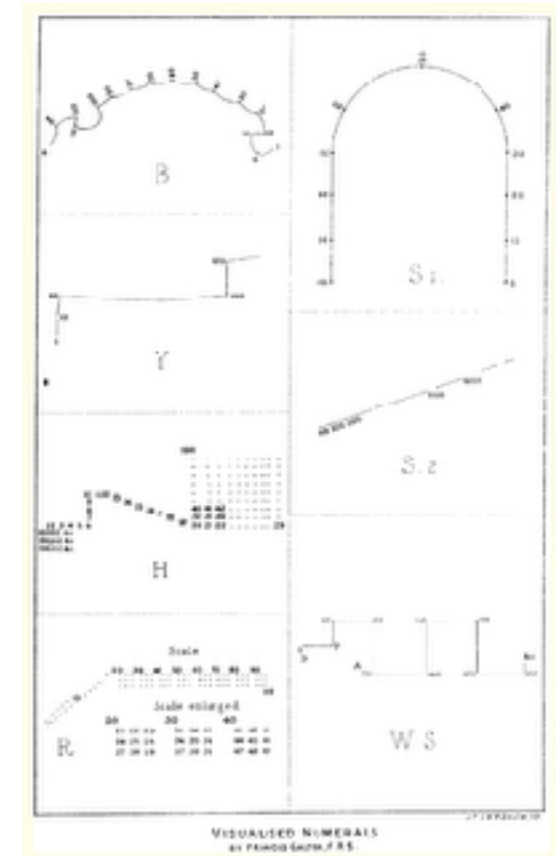
Mathieu, Epinat-Duclos, Sigovan, Breton, Cheylus, Fayol, Thevenot and Prado (2018) Cereb. Cortex

Numbers in space

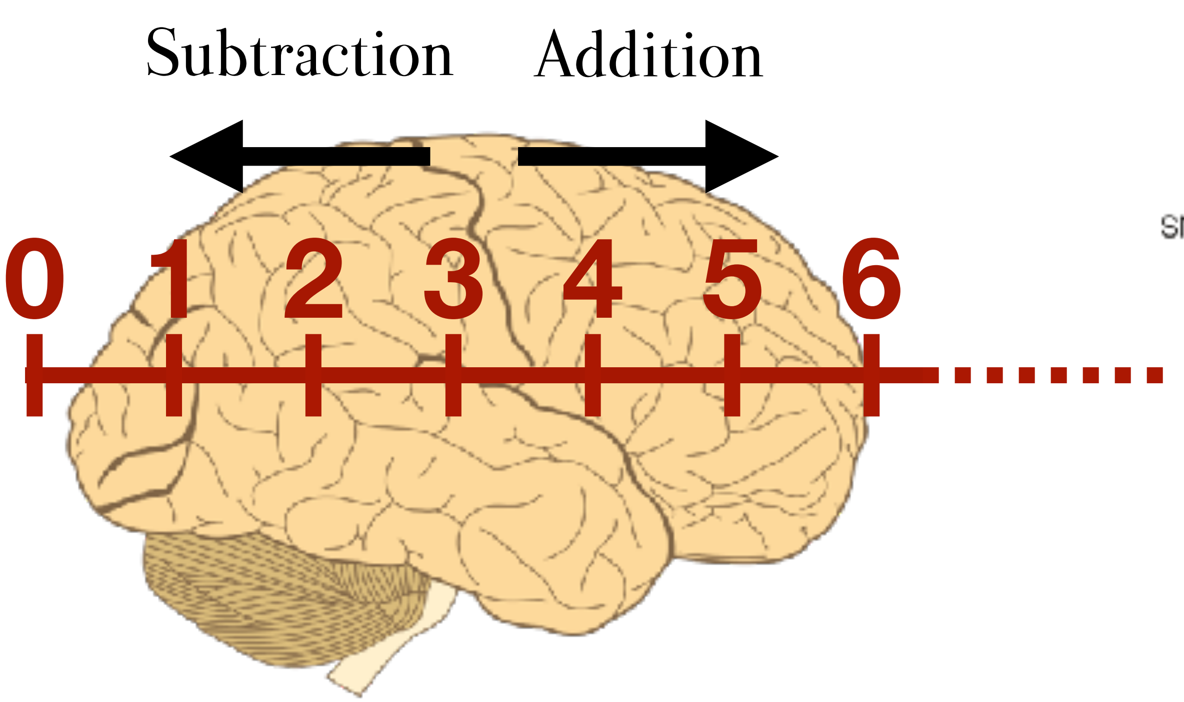
- There is evidence of explicit and implicit associations between numbers and space (i.e., the mental number line)

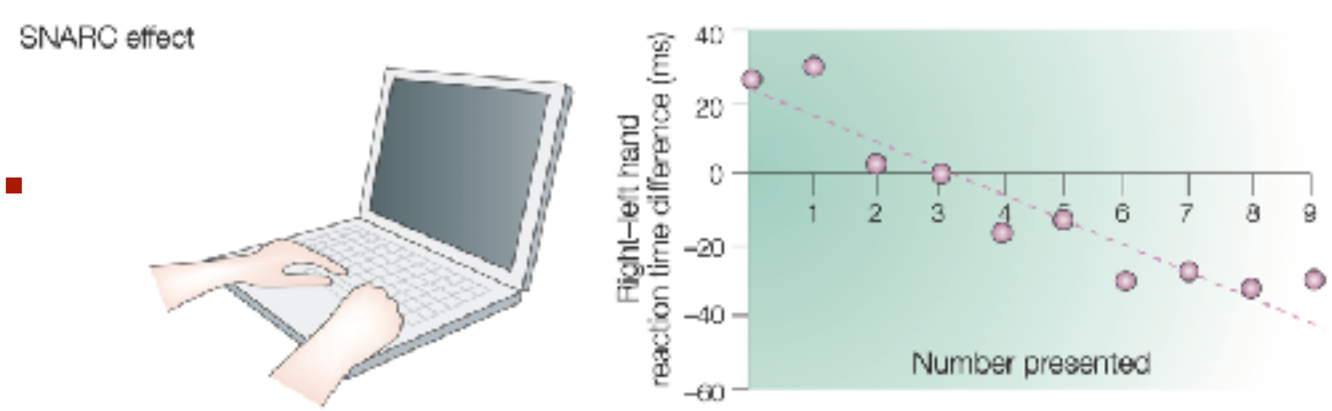
- Adding and subtracting may involve navigating along the mental number line





Galton (1881)

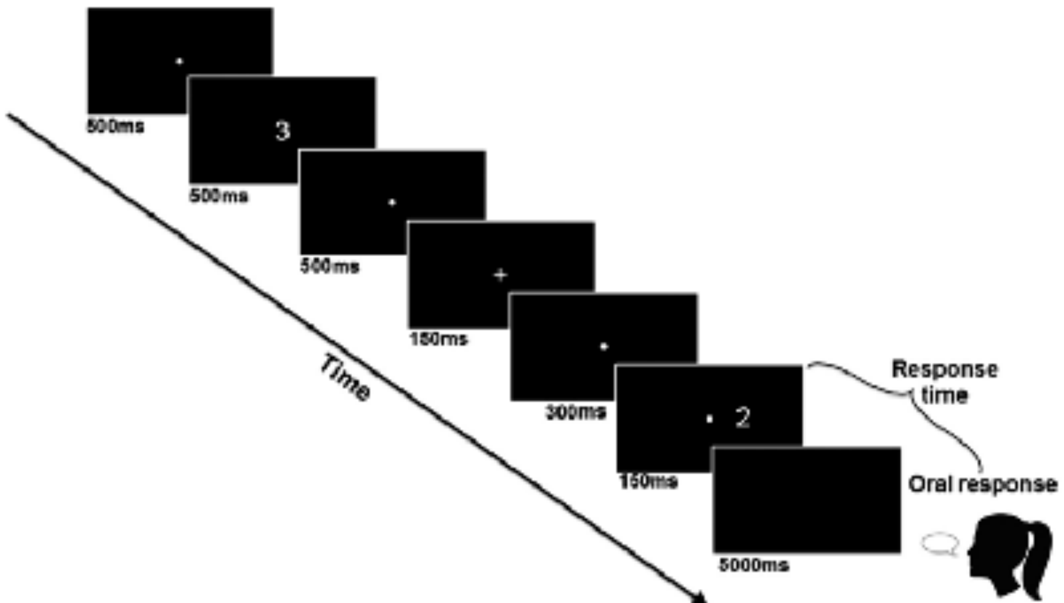




Dehaene, Bossini, & Giraux (1993) JEP: Gen.

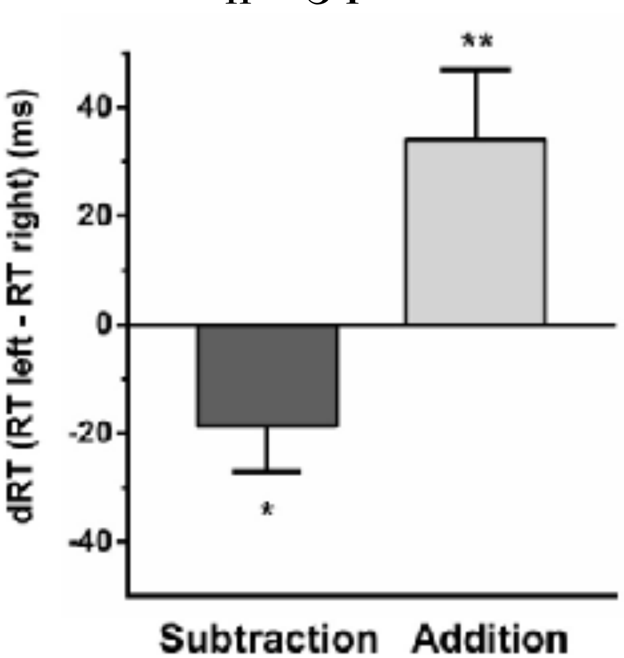
Operator processing

A. Díaz-Barriga Yáñez

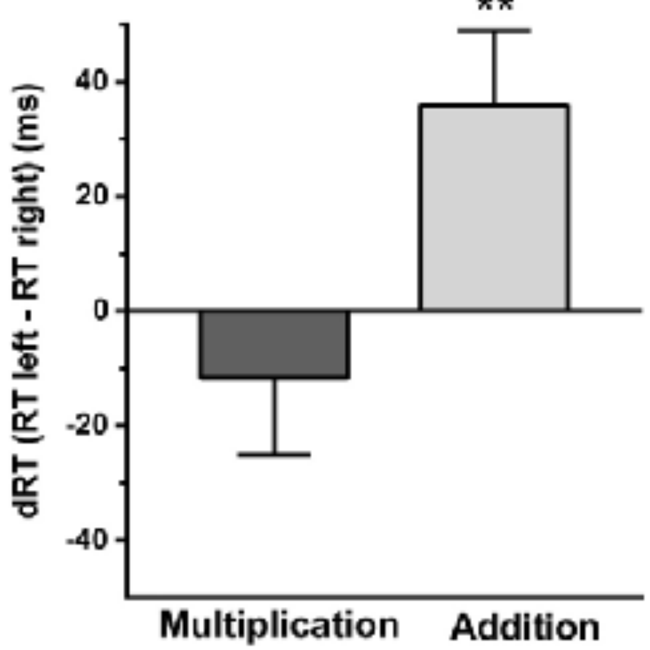


Adults

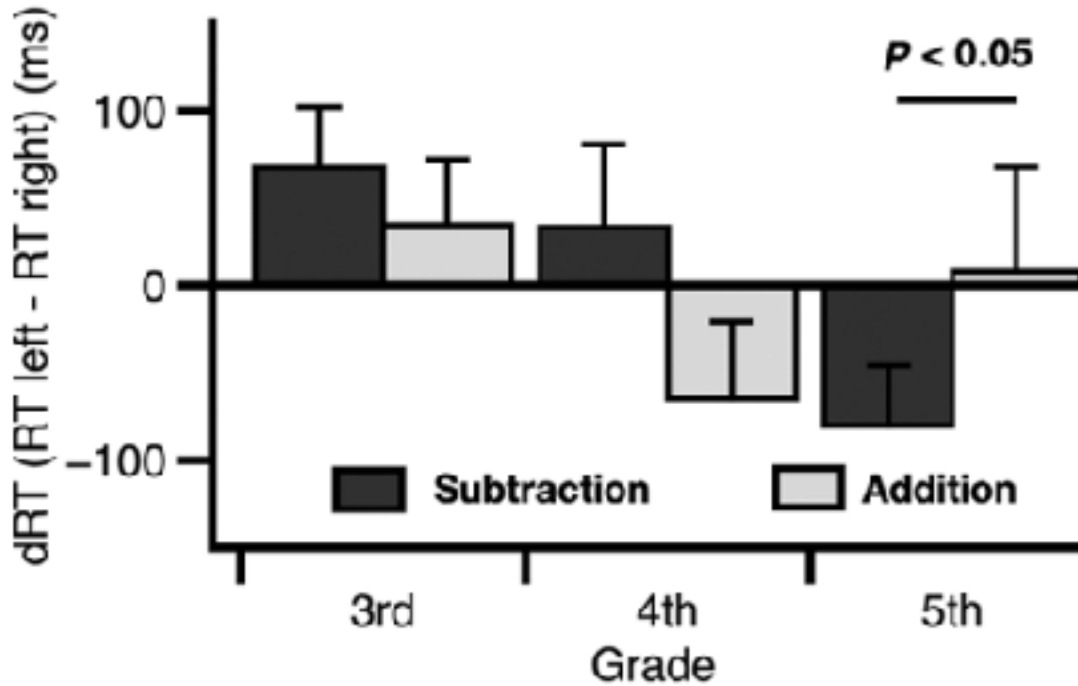
n = 34



n = 22



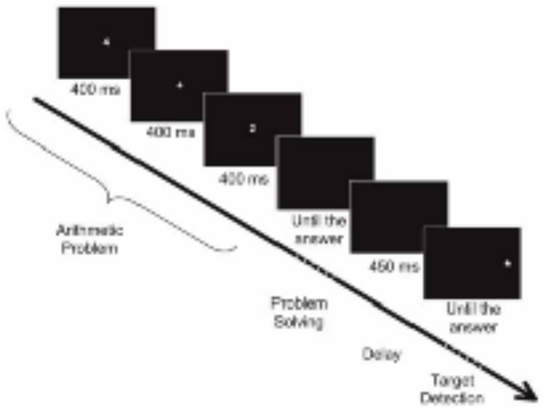
Children (n = 101)



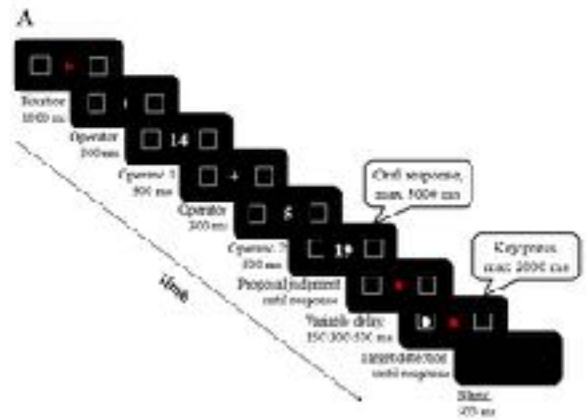
Mathieu, Gourjon, Couderc, Thevenot, & Prado (2018) Cognition
Díaz-Barriga Yáñez, ..., Thevenot and Prado (2020) Ann. N. Y. Acad. Sci

Arithmetic and space

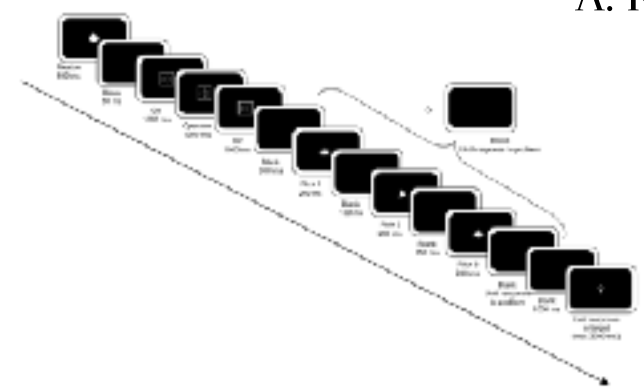
A. Knops



Masson & Pesenti (2014)

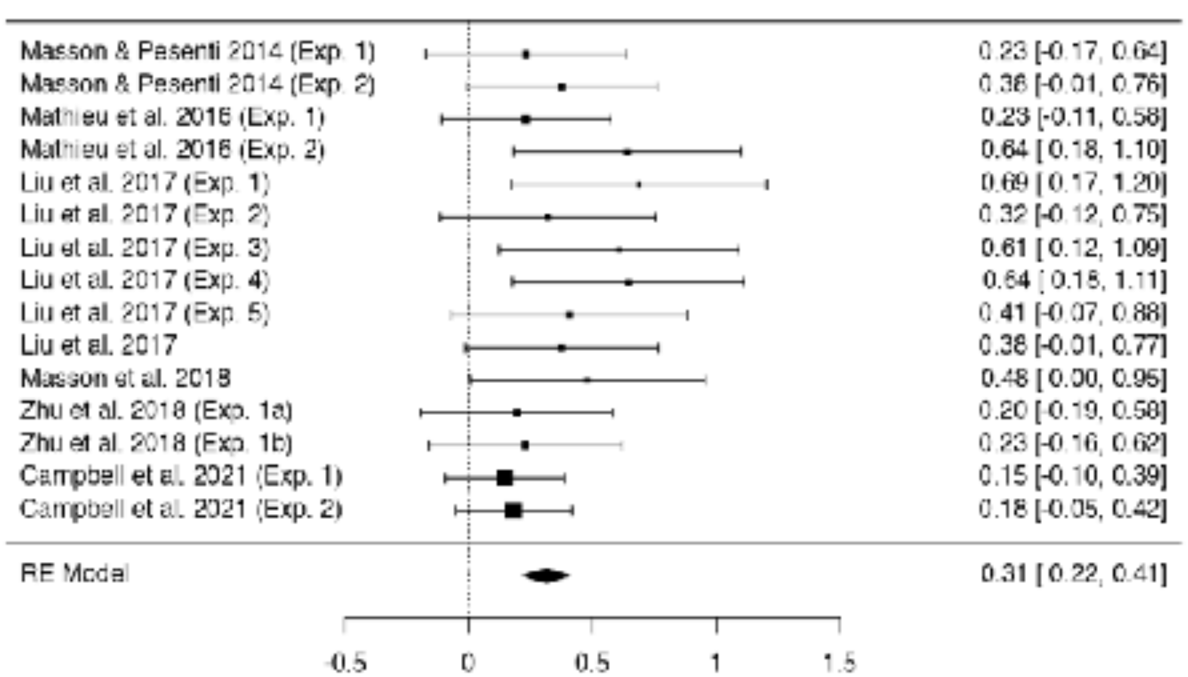


Liu et al. (2017)

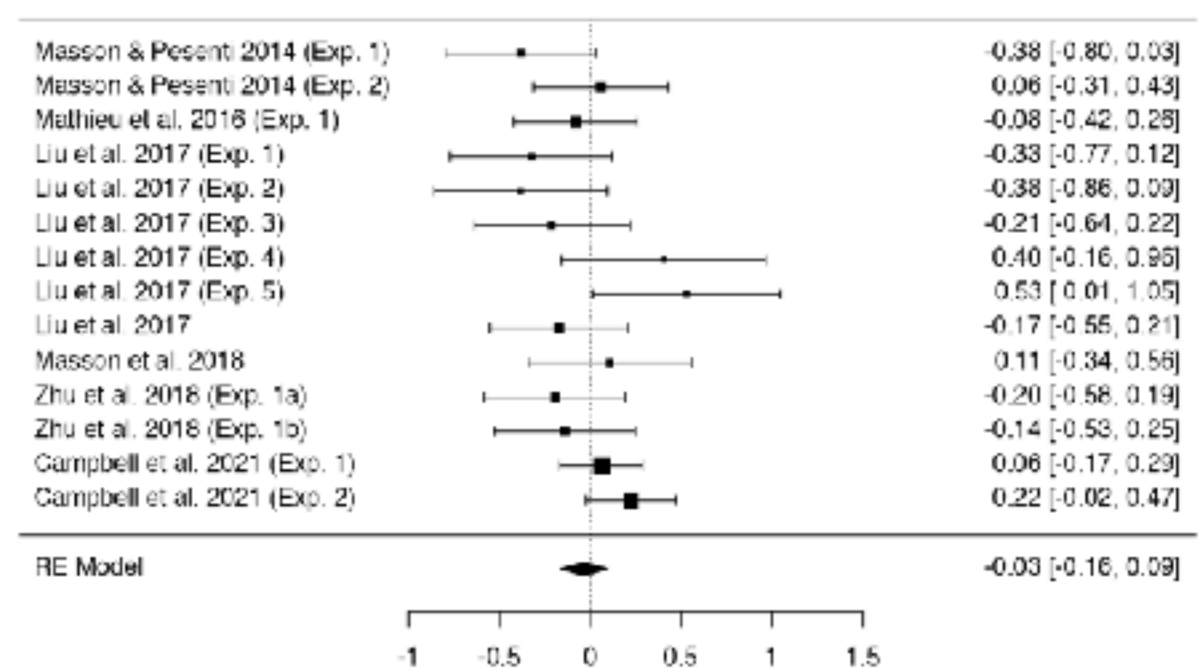


Masson & Pesenti (2015)

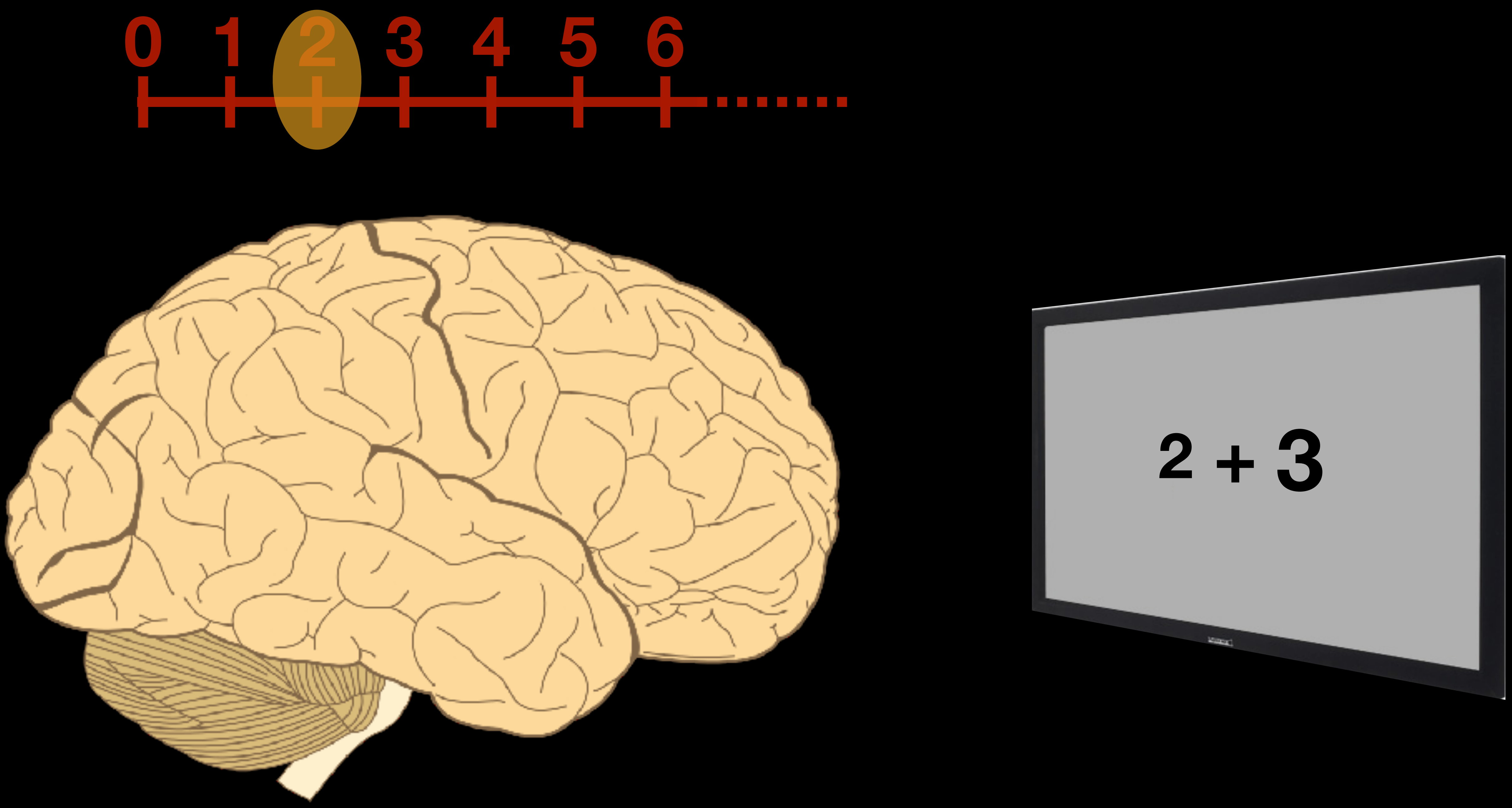
Addition

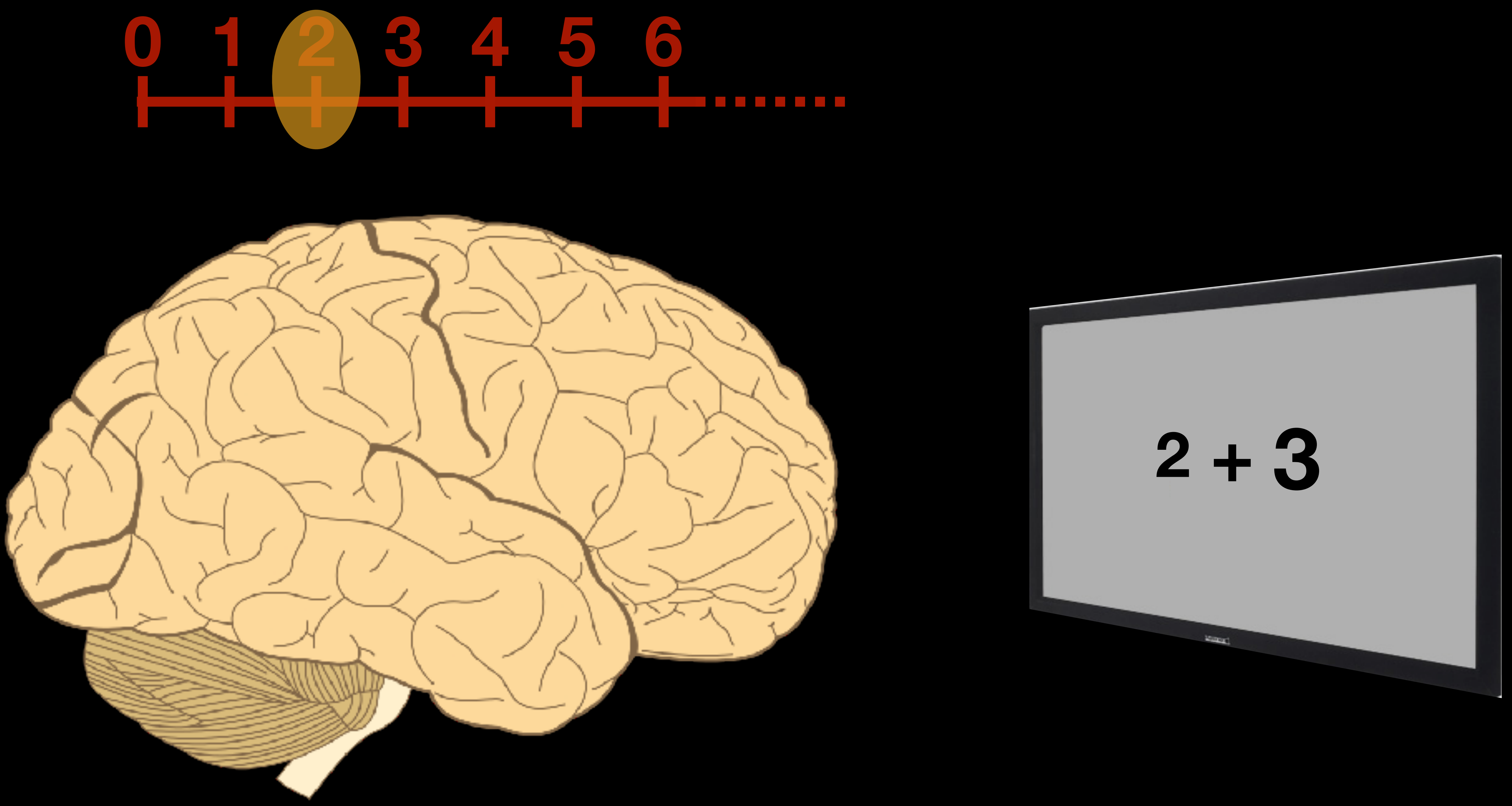


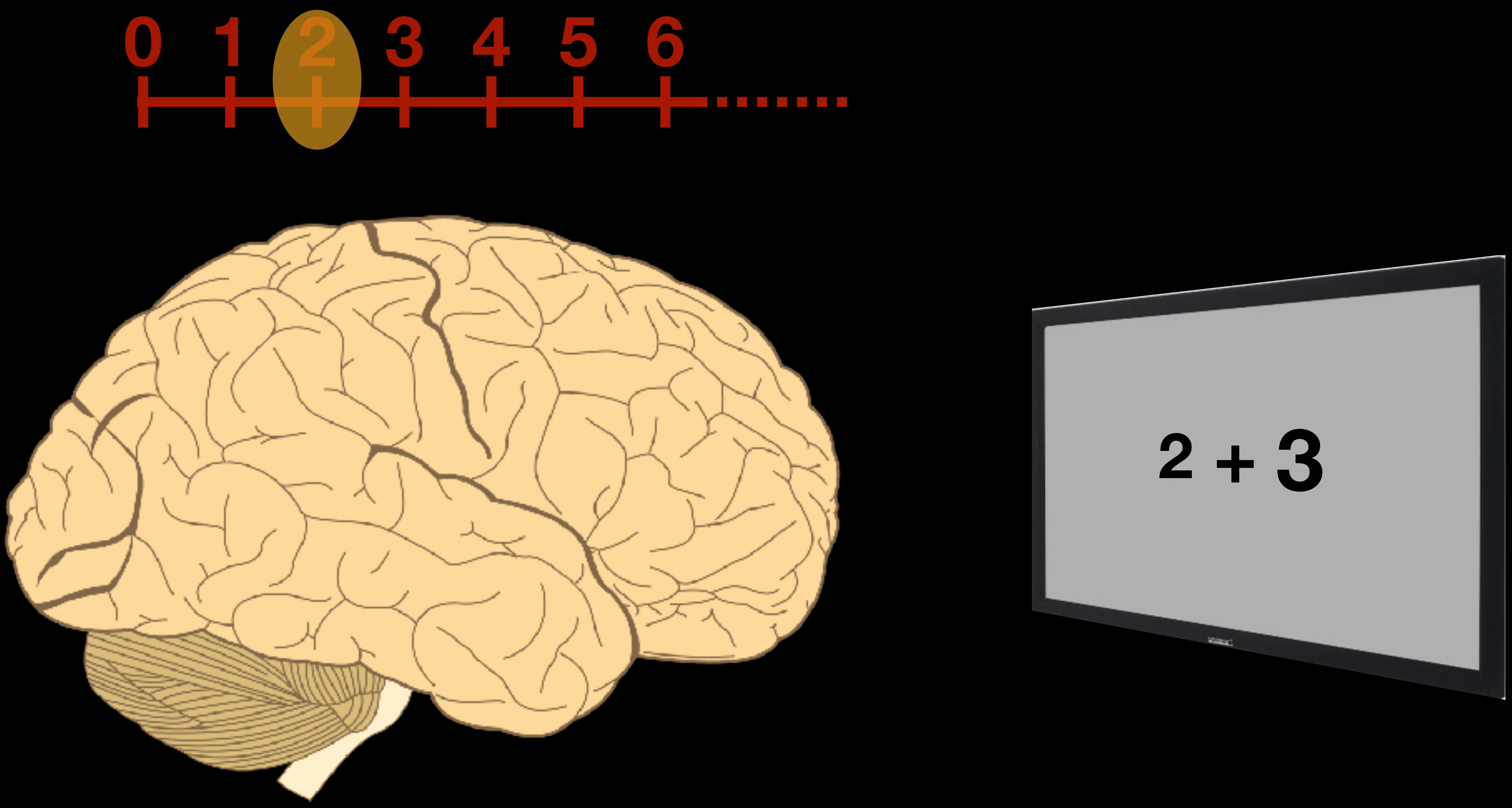
Subtraction

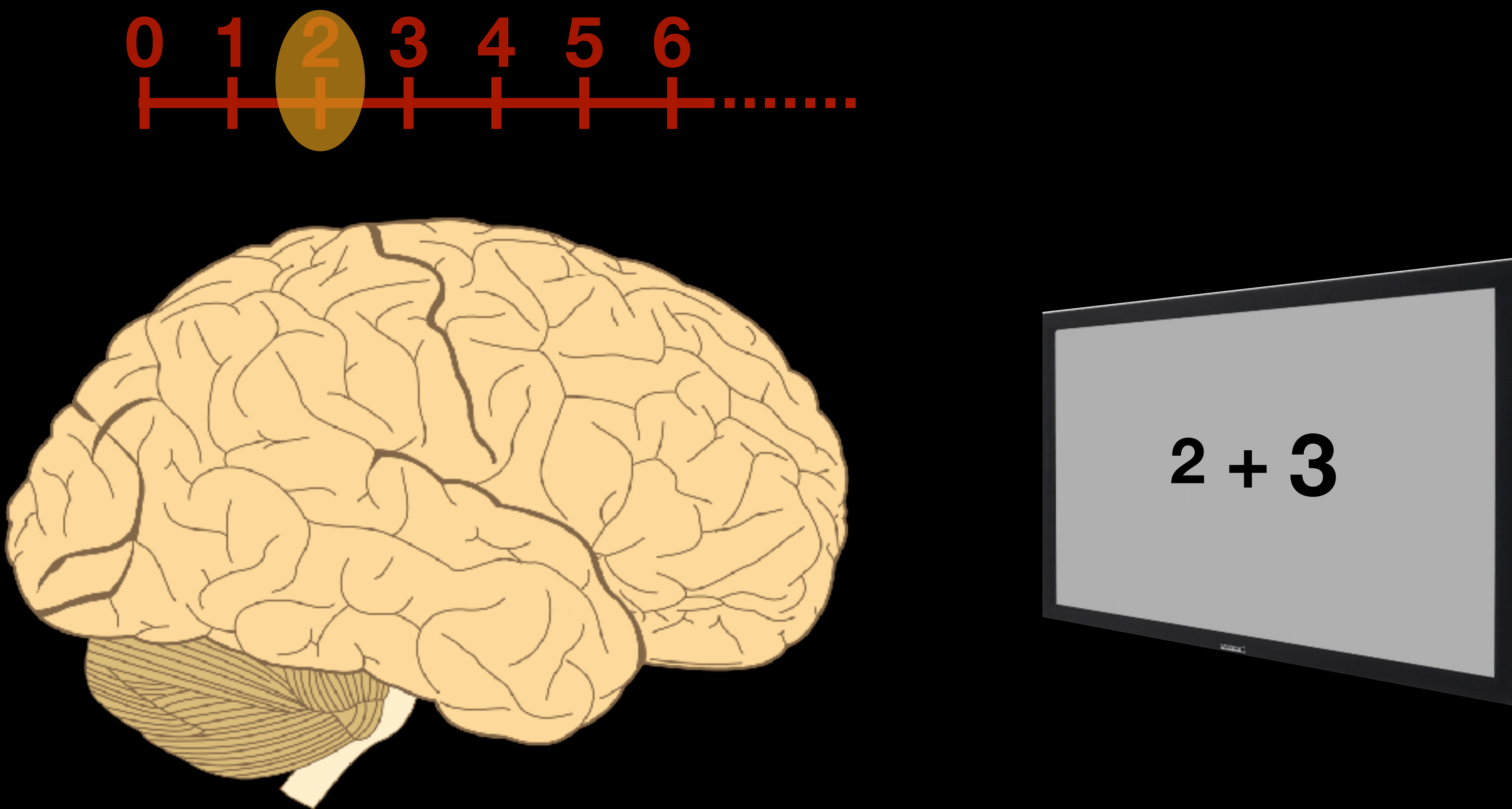


Prado & Knops (in revision)


0 1 2 3 4 5 6
2 + 3


0 1 2 3 4 5 6
2 + 3


0 1 2 3 4 5 6
2 + 3


0 1 2 3 4 5 6
2 + 3

Dissecting arithmetic facts

$$2 + 3 = 5$$

$$2 + 3 = 5$$

$$\color{red}{2} + \color{red}{3} = \color{red}{5}$$

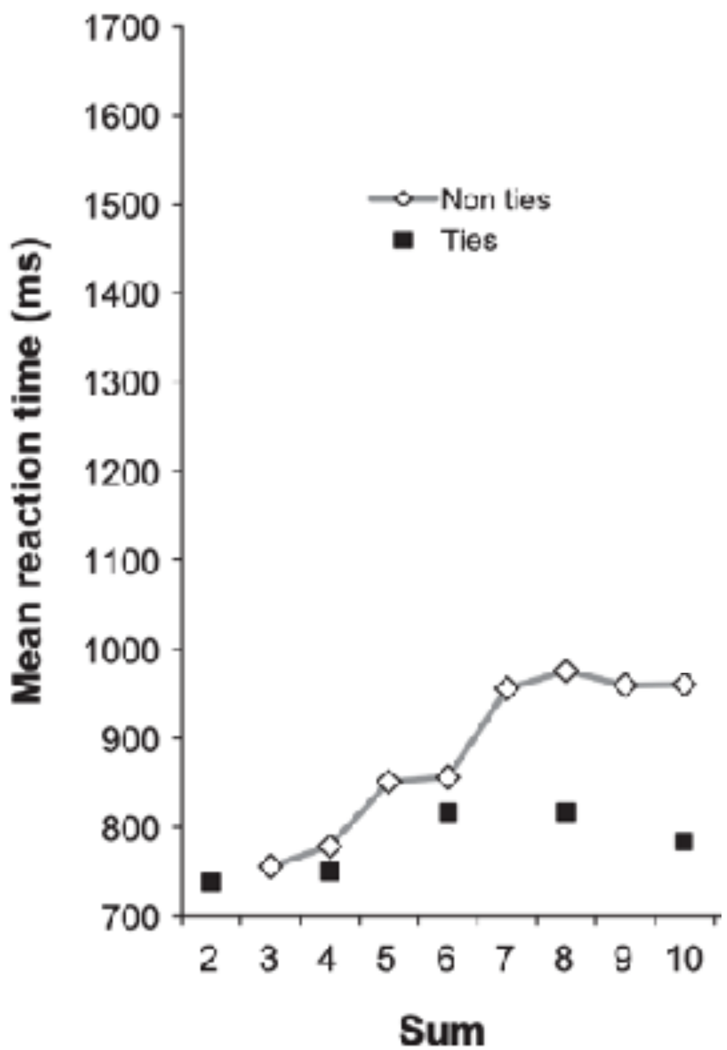
The problem-size effect

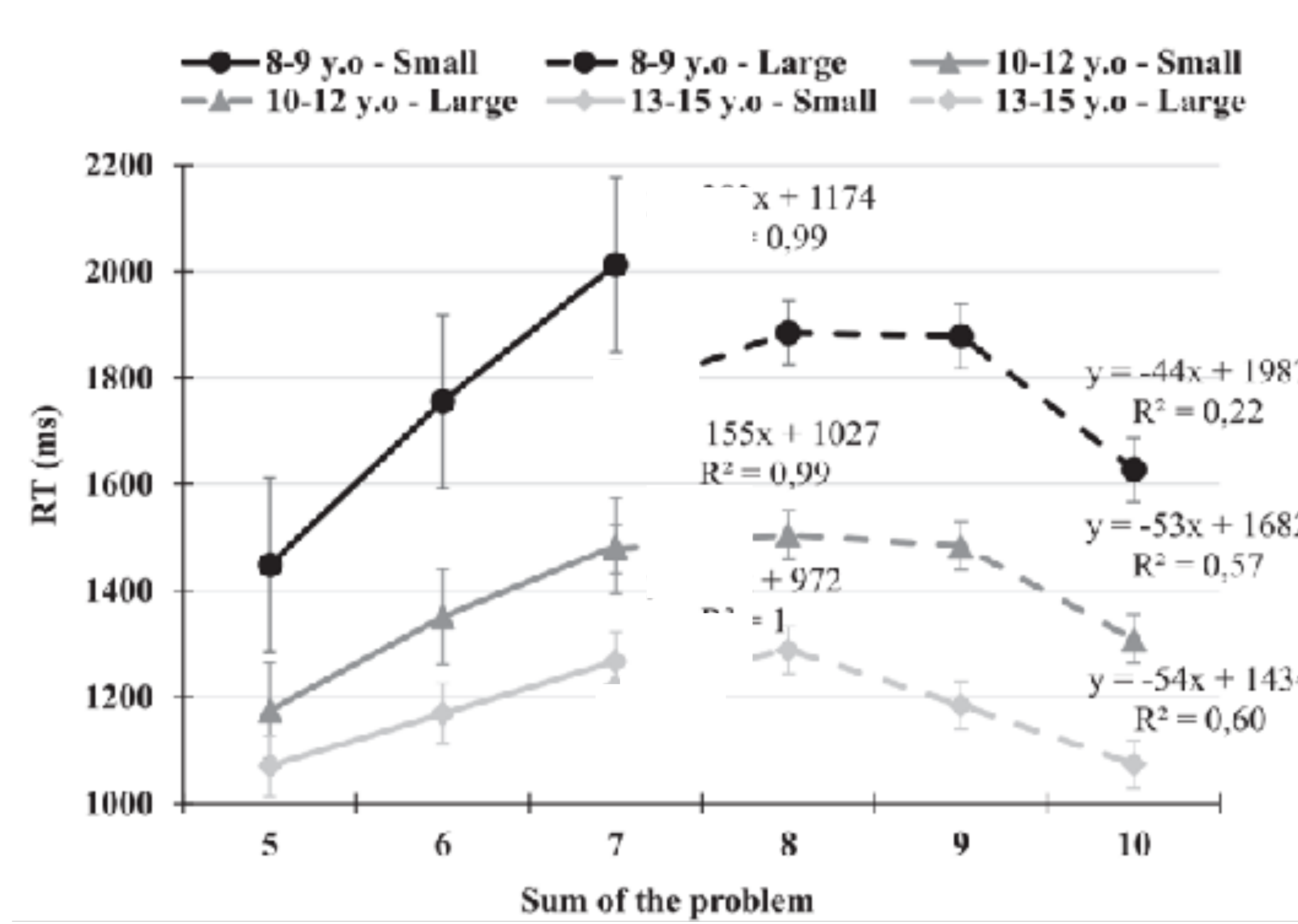
C. Poletti

2 + 3
2 + 4
3 + 2
3 + 4
4 + 2
4 + 3

Adults (n =90)

- 8-9-yo (n = 39)
- 10-12-yo (n = 42)
- 13-15-yo (n = 24)





Poletti, Díaz-Barriga Yáñez, Prado, & Thevenot (2023) J. Exp. Child Psych.
Uittenhove, Thevenot, & Barrouillet (2016) Cognition

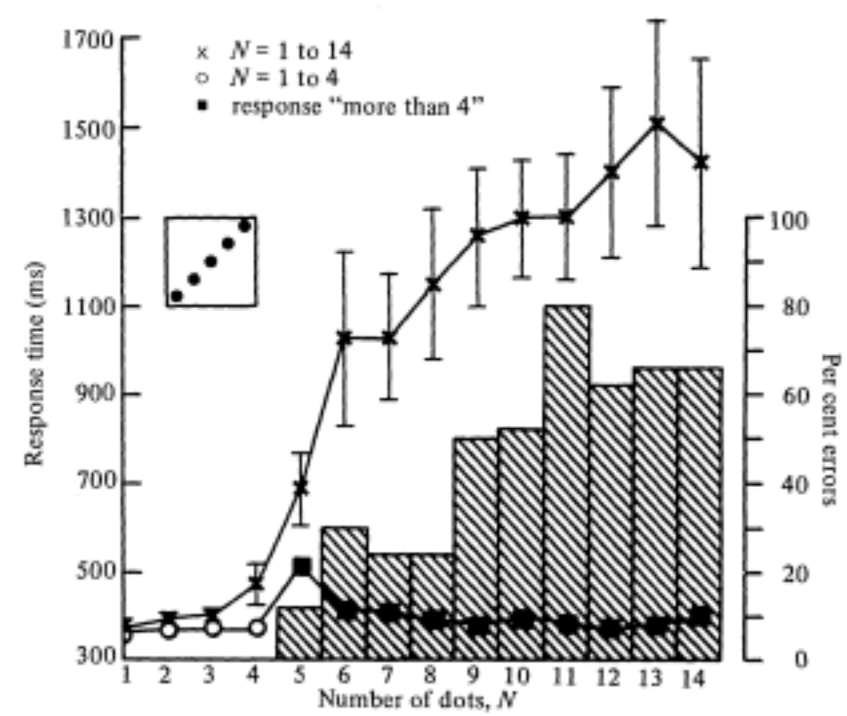
The ‘magical’ number 4

What is Magic About the Magical Number Four?

Dietrich Simons and Dietrich Langheinrich

The magic number 4 ± 0: A new look at visual numerosity judgements

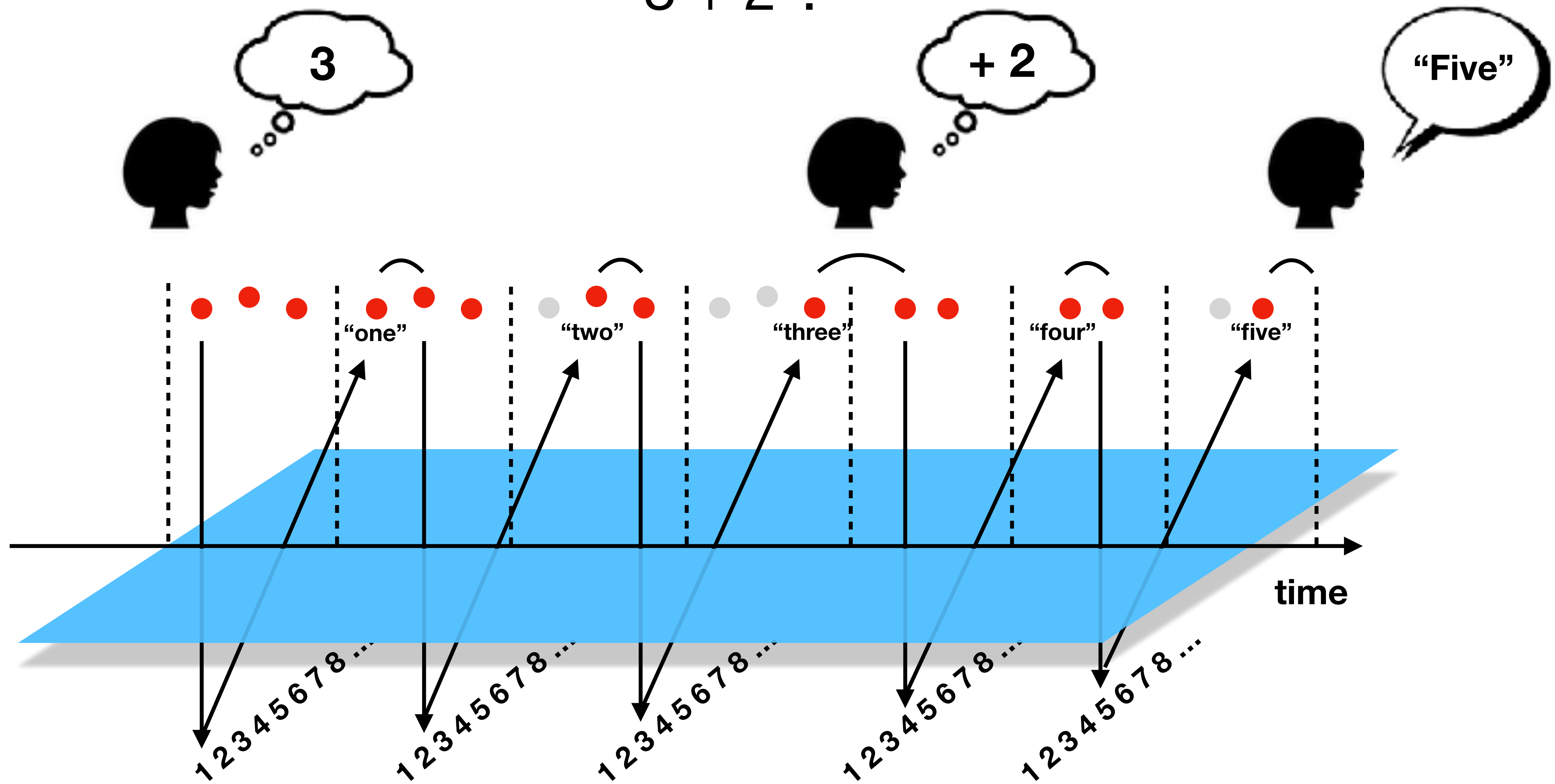
Janette Atkinson, Fergus W Campbell, Marcus R Francis



Atkinson, Campbell, & Francis (1976) Perception

An automatized counting model

3 + 2 ?



1. Brain regions in which activity is associated with the problem-size effect in children should still contribute to the problem-size effect in adults

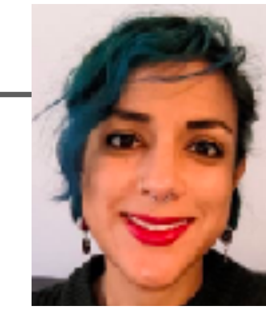
2. BUT this should be limited to problems with operands ≤ 4

Uittenhove, Thevenot, & Barrouillet (2016)

Neural development of the problem-size effect

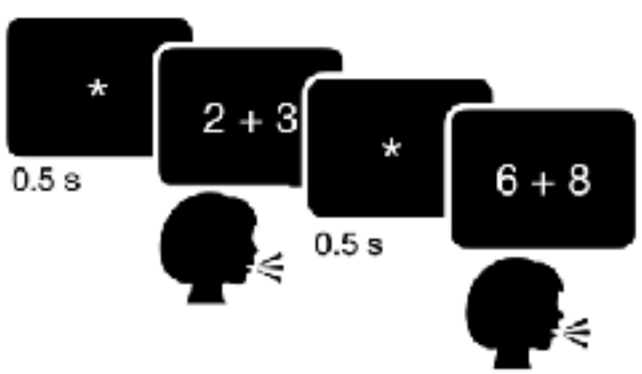
- n = 128 participants

- 8-9-yo (n=31)
- 11-12-yo (n=31)
- 14-15-yo (n=26)
- Adults (n=40)

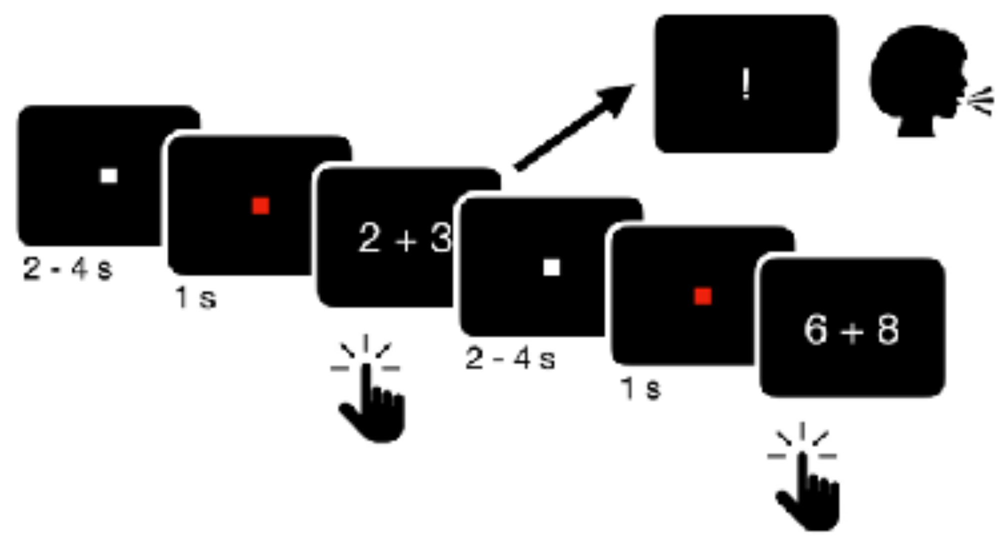


A. Díaz-Barriga Yáñez

<u>Vocal production (out-of-scanner)</u>



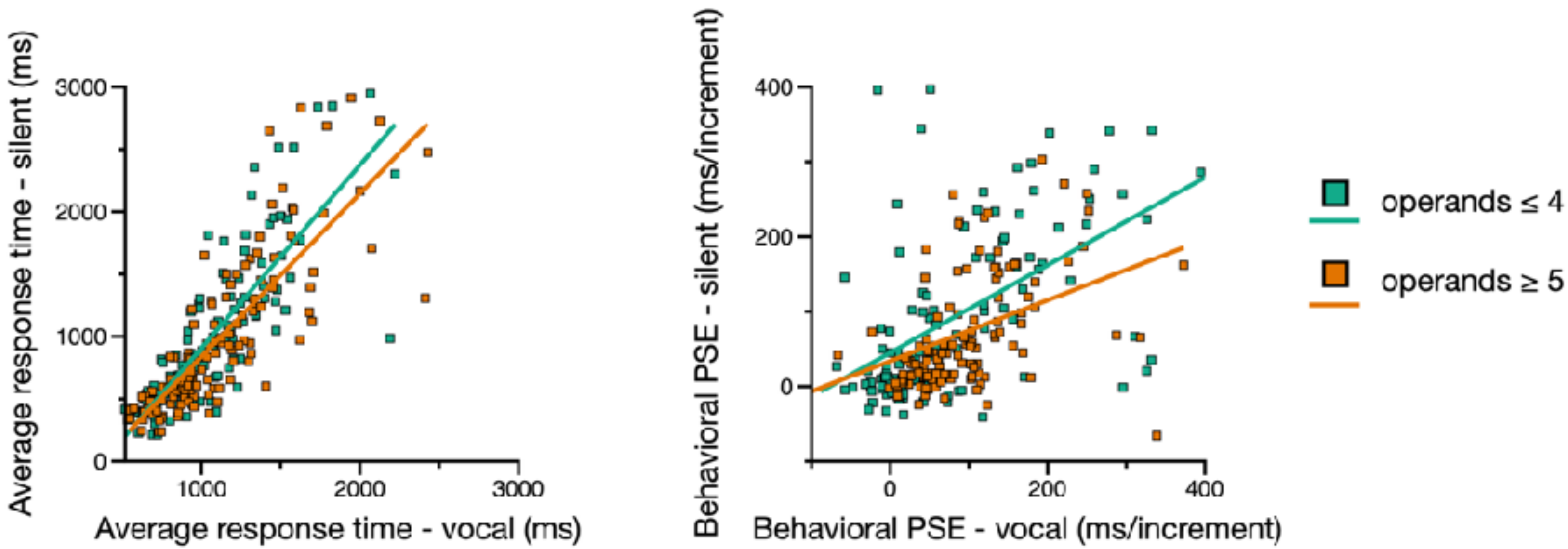
<u>Silent production (in-scanner)</u>



Díaz-Barriga Yáñez, Longo, Chesnokova, Poletti, Thevenot, & Prado (in revision)

Neural development of the problem-size effect

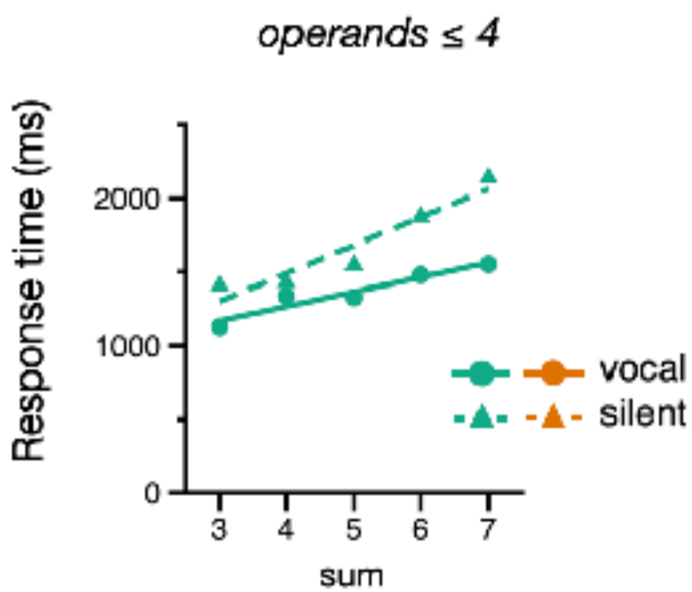
Vocal (out-of-scanner) vs. Silent (in-scanner) task

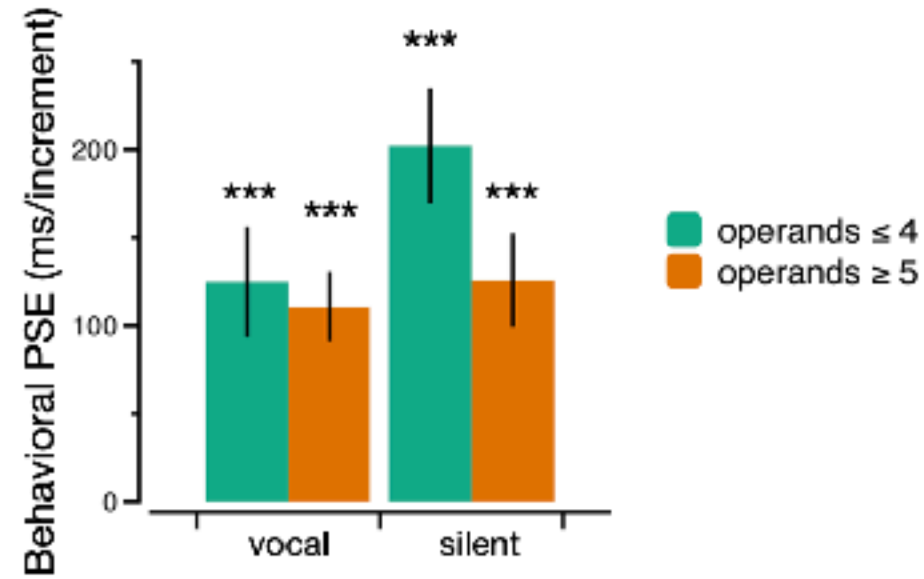


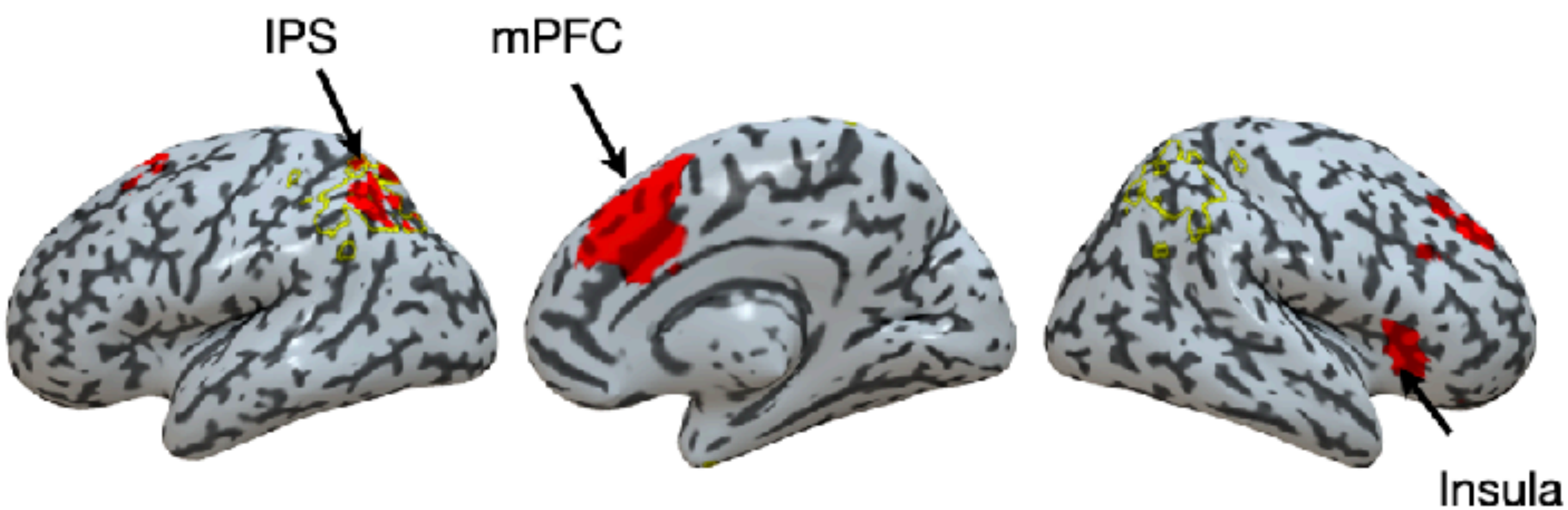
Díaz-Barriga Yáñez, Longo, Chesnokova, Poletti, Thevenot, & Prado (in revision)

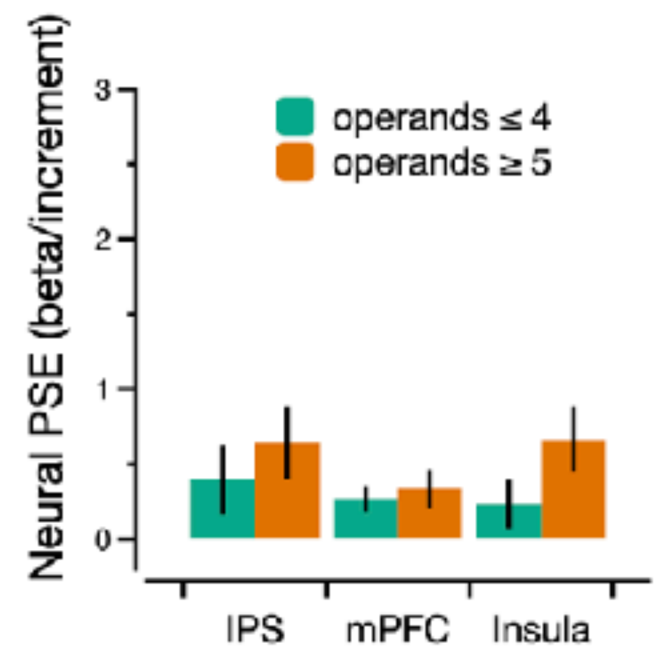
Neural development of the problem-size effect

<u>Participants younger than 10 (n = 31)</u>







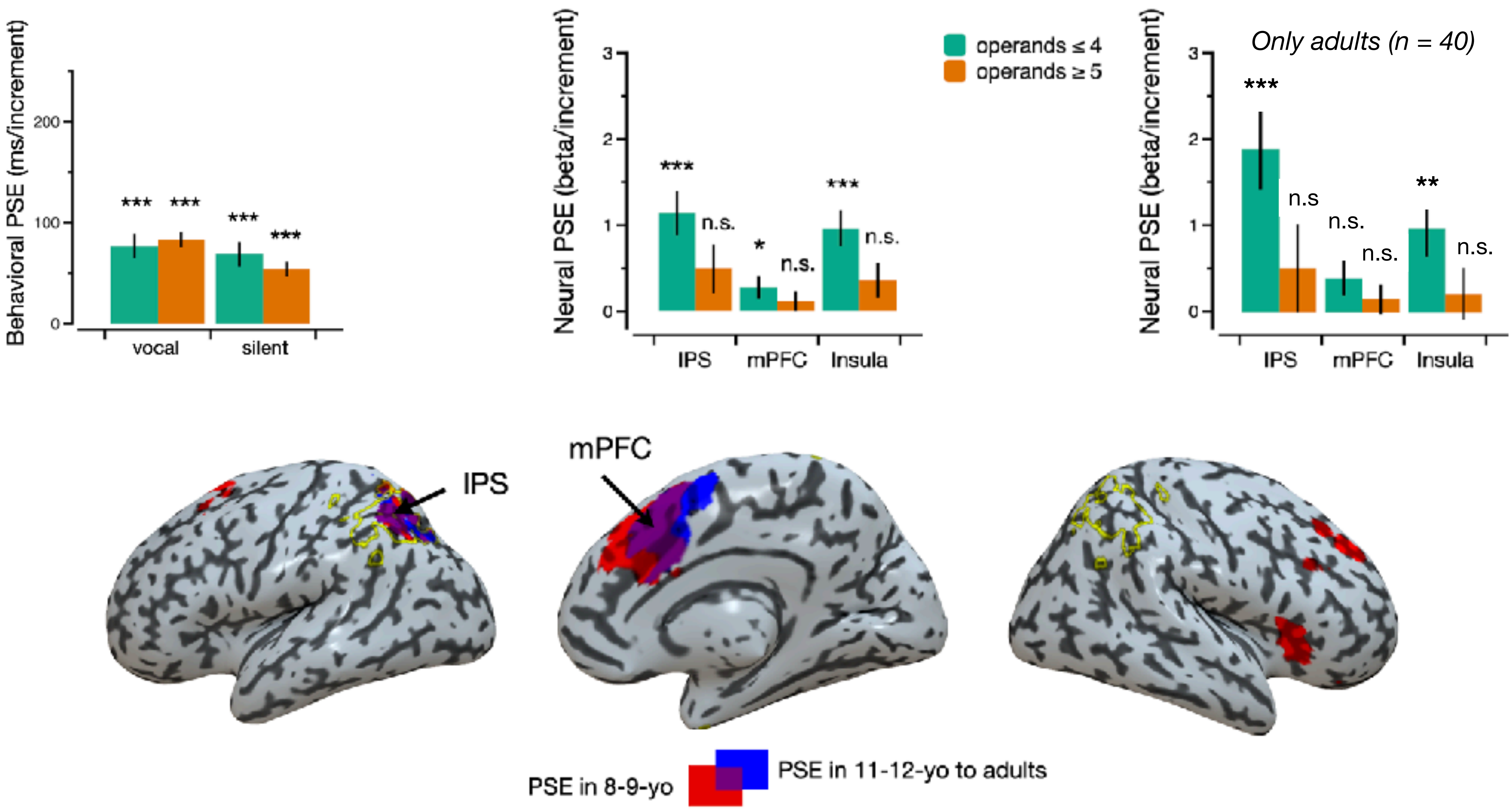


3 counting-related ROIs

Díaz-Barriga Yáñez, Longo, Chesnokova, Poletti, Thevenot, & Prado (in revision)

Neural development of the problem-size effect

Participants older than 10 (n = 97)



Díaz-Barriga Yáñez, Longo, Chesnokova, Poletti, Thevenot, & Prado (in revision)

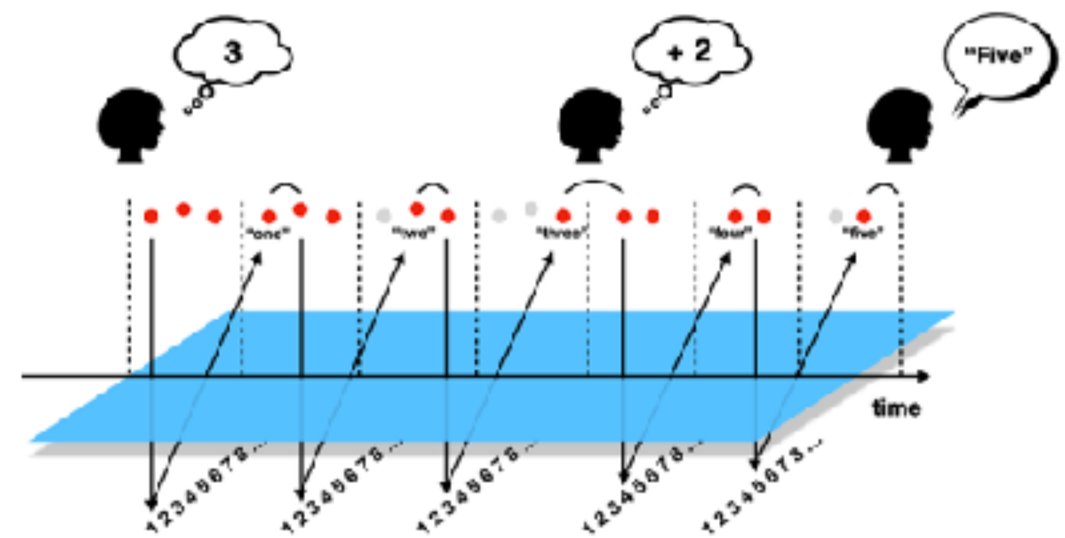
Neural development of the problem-size effect

- The problem-size effect decreases over the course of learning, but remains significant even in expert adults and even in very small problems

- Neuroimaging evidence suggests that qualitatively similar neural mechanisms support the problem-size effect in children and adults, though this is limited to problems with operands ≤ 4

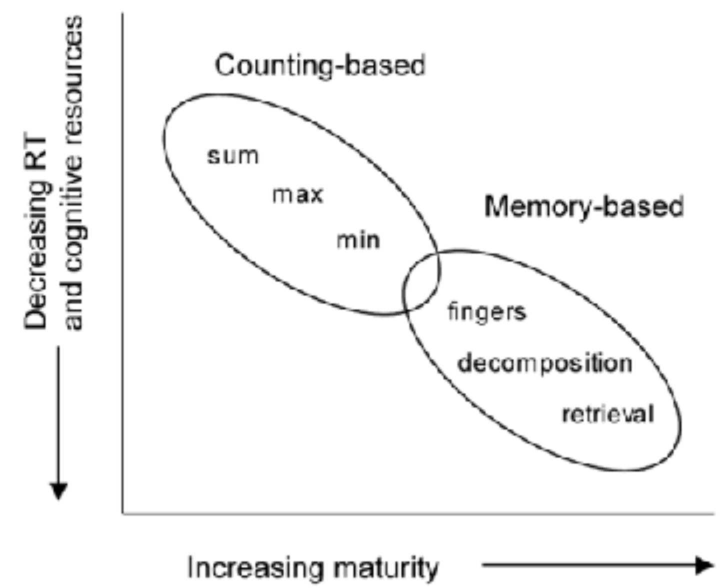
- This is consistent with the idea that a counting procedure may be automatized over the course of development

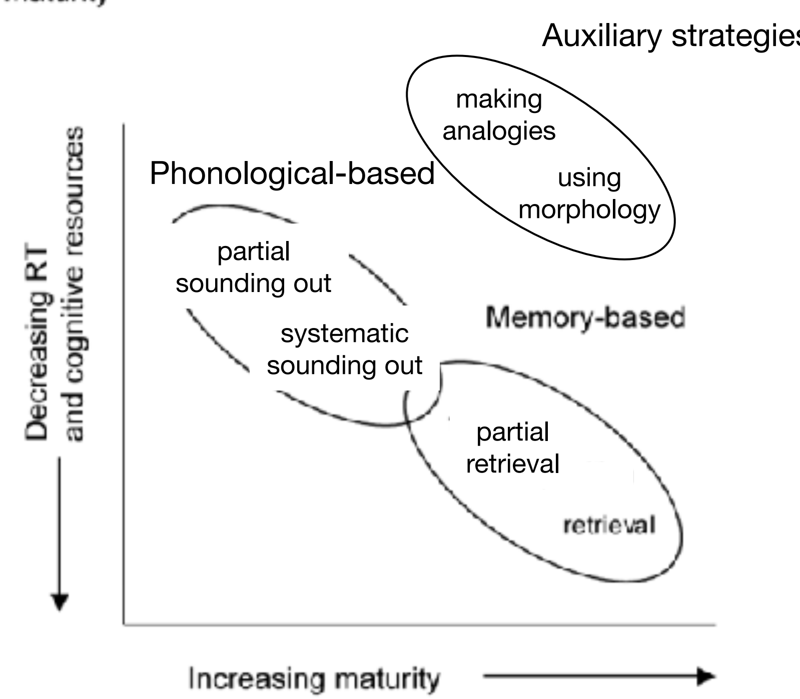
- This procedure may involve a tagging of numbers along a mental number line, leading to an association with space



Theoretical Implications

- Memory may be largely associative, but that does not mean that learning to solve frequently encountered problems will necessarily rely on building associations in children
- Learning also involves overly practicing procedures, which may increase in efficiency and become automatic and unconscious in adults
- Self reports cannot distinguish between automatized procedures and associations
- Automatized procedures and associations may compete in a "horse race"
- This has implications for other academic domains

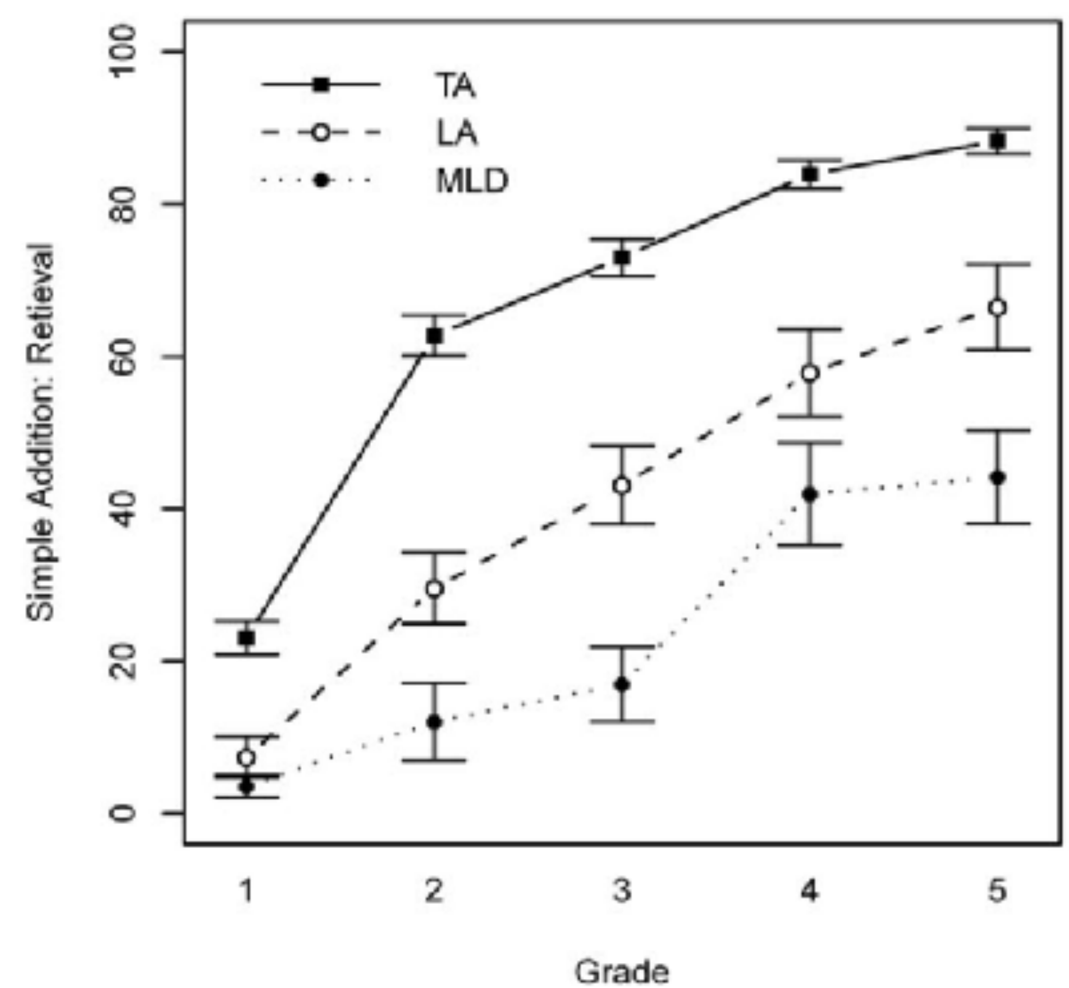




Farrington-Flint Coyne, Stiller & Heath (2008) Educ. Psych.

Clinical implications

- Math learning disability (MLD) affects 5-6% of children worldwide
- A hallmark of MLD is a persistent inability to fluently process arithmetic facts
- This inability is often interpreted as a retrieval deficit due to working memory limitations
- However, MLD might also involve inefficient automatization of procedures
- This is consistent with the procedural deficit hypothesis of learning disabilities



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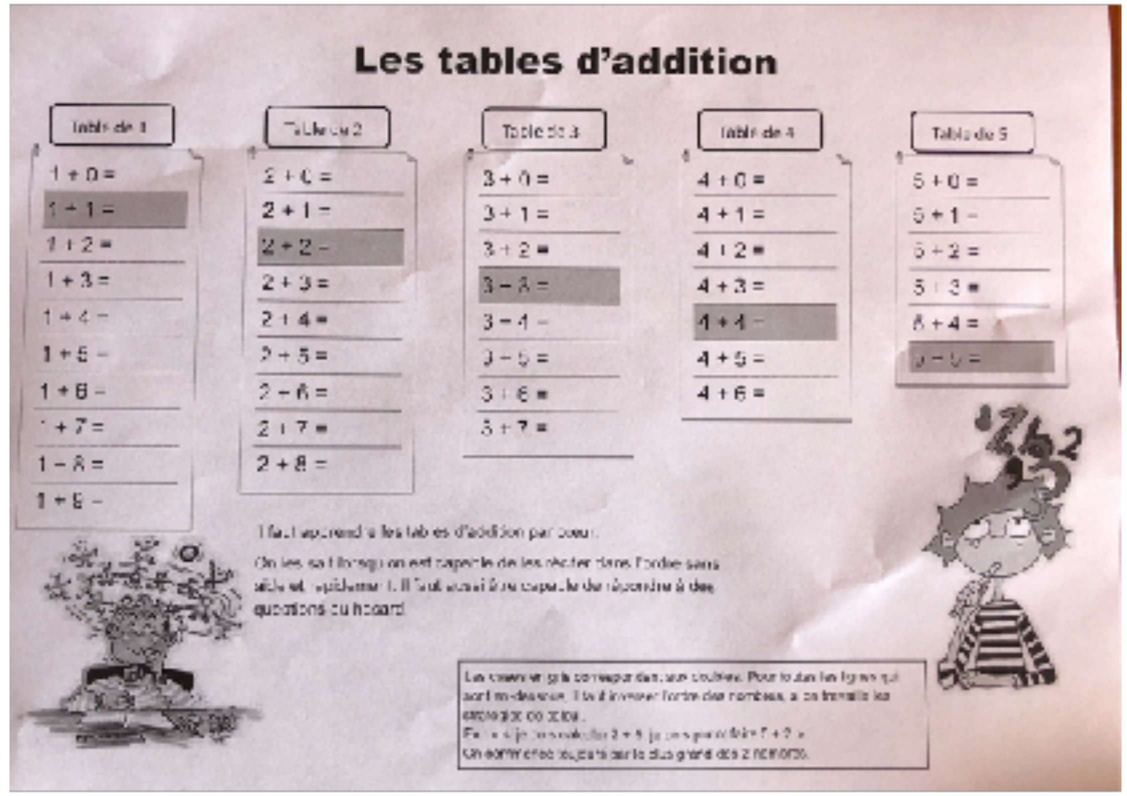
Educational implications



- There is no doubt that building fluency with arithmetic facts is important
- However, arithmetic facts do not necessarily need to be learned by rote, which comes at a cost of interferences
- Practicing a procedure may be as effective as rote learning, to the extent that its application is straightforward and it is sufficiently practiced
- Multiplication tables make sense, addition tables much less

Thanks

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The BBL team

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anr agence nationale de la recherche

MARIE CURIE ACTIONS

Fondation de France